Schillinger's Special Theory of Harmony: Hybrid 5- and 4-part harmony

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Abstract

This document describes two techniques as presented by Schillinger [1] in his *Special Theory of Harmony* book. Chord structures, progressions and application aspects of the hybrid 5- and 4-part harmony technique will be discussed and illustrated with score examples.
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1 Introduction

Schillingers Book 5: *The Special Theory of Harmony* [1] presents a great summary of tonal 4-part harmony. Tonal harmony implies that chord structures and progressions are based on a ordering in thirds of seven-pitch diatonic scales (Schillinger calls this the first *expansion* form of the scale). This means that if we write a 7-pitch scale on a tonic p_1 as an ordered set (ascending diatonic steps of a second) $E_0 = \{p_1\, p_2\, p_3\, p_4\, p_5\, p_6\, p_7\}$, then the first expansion (into a sequence of thirds) is $E_1 = \{p_1\, p_3\, p_5\, p_7\, p_2\, p_4\, p_6\}$. Chord structure is determined by neighbouring subsets: e.g., a triad is formed by taking 3 neighbouring pitches, such as $\{p_1\, p_3\, p_5\}$ (the tonic triad chord) or $\{p_5\, p_7\, p_2\}$ (the dominant triad chord). Root progressions are determined by pitch pairs and are more natural when the pair lies close together and the movement is from right to left (from last to first pitch): e.g., root movement by a third downward from $p_3 \rightarrow p_1$ is more natural than $p_6 \rightarrow p_2$ (a fifth downward) or a third upward $p_5 \rightarrow p_7$[1].

The *hybrid 5- and 4-part* techniques are extensions to the standard practice in tonal harmony. These will not be found in other textbooks. The hybrid 5-part technique is discussed in Schillinger's book after the chapter on the seventh chord; as we will see below it combines the treatment of the dissonant 7th with the *positive root cycles*. The hybrid 4-part technique involves a generalisation of voice leading in triadic chord progression with positive cycle root movement.

The hybrid 5-part technique allows the use of *extended chords* with acceptable voice leading, while preventing the complete parallel voice movement, so typical of *sectional harmony settings* in jazz big band scores.

Since the availability of the Schillinger books is limited, this document presents an overview of these techniques with comments and new examples.

2 Notation

We will use the following notation:

[1] Note that this concept deviates from conventional theory of harmony, where the dominant to root (a fifth down) chord progression is the basis for chord cadences.

Table 1: Hybrid 5-part chord structures

Upper	5	7	9	11	13
Structure	3	5	7	9	11
S_7	1	3	5	7	9
	13	1	3	5	7
Root	1	1	1	1	1
Chord structure	S_5	S_7	S_9	S_{11}	S_{13}

$p[i]$: i-th pitch from a chord structure, with $i = 1, \ldots, i_{max}$ (any pitch from a chord structure with i_{max} pitches).

$j = \{1, 3, 5, 7, 9, 11, 13\}$: the *function* of a pitch in a chord structure. Note that the function group is based on the ordering in thirds of a seven-pitch diatonic scale $j = 1$ is the *root* of the chord, $j = 3$ the third, $j = 5$ the fifth, etc.

S_k: chord structure with $k = \{5, 7, 9, 11, 13\}$ indicating the *tension* of the chord. The tension is determined by the highest function pitch in the chord structure. E.g., S_5 is a pure triad, S_7 a 7th chord, S_9 a 9th chord, etc. Special numbers indicate chord *inversions*: S_6 is the first inversion, S_4^6 the second inversion of a triad S_5. S_4^6 the first inversion, S_3^4 the second inversion and S_2 the third inversion of a seventh chord S_7.

R_l, with $l = \{\pm 3, \pm 5, \pm 7\}$, indicates the *root cycle*, i.e., the root movement in a *chord progression* (going from one chord to the next). R_3 means the root is moving a third downward, R_{-3} means a root movement a third upward, etc.

3 Hybrid 5-part harmony

3.1 Chord structure

Valid hybrid 5-part chord structures are shown in Table 1. The full chord structure is split into an upper and lower structure, that will be treated differently during chord progressions. Note that the upper structure is an S_7 (4 neighbouring pitches from the first expansion of the diatonic scale, yielding a dissonant 7th interval between the lowest and upper function). The S_5 in fact is a triad with added 6th or 13th[2], i.e., S_5^{add6}. Note that the intervals of the third between functions are not specified, so all kinds of S_5 (major, minor, augmented or diminished triad), S_7 etc. are possible.

What is new in this extended technique is the fact that harmonic continuities, i.e., sequences of these extended chords may be used. This is different from traditional tonal harmony where sequences of S_7 structures are the maximum tension string allowed; higher tension chords (S_9 and S_{11}) may occur only as single, isolated chords (the must be properly prepared and followed by lower tension chords).

3.2 Upper structure voice leading

The rules for the upper structure voice leading are:

[2]The added 6th is a traditional and frequently used extension to the tonic or subdominant triad.

1. At each chord progression the dissonant upper function of the chord (i.e., the 7th from the upper S_7 structure) is properly resolved by downward stepwise movement or slurred into a common note in the next chord[3]. The need for proper preparation of the dissonant 7th is released[4].

2. The lower three voices of the upper structure are free to move in any direction. However, in practice stepwise movement or small leaps are preferred. Crossing of voices is avoided (although not prohibited; we will see occasional voice crossing in the examples). If all voices move downward with minimal movement, the result will look like jazz music sectional harmony, but in general we now have more freedom in independent voice leading.

3.3 Root movement

The root (lower structure) movement is determined by the following rules:

1. The lower structure has a constant function, i.e., chord root.

2. The root moves preferrably according to positive root cycles, i.e., R_3 (3rd down or 6th up), R_5 (5th down or 4th up) or R_7 (7th down or, more likely, 2nd up). Occasionally negative root cycles (R_{-3}, R_{-5} and R_{-7}) may be used.

3.4 Application and examples

Chord progressions of hybrid 5-part harmonies can be used in any of the three *forms of harmonic continuity*, that Schillinger discerns:

1. Type I: diatonic chord structures and root movements. Schillinger remarks that the scale used for building the chord structures and the root progression need not necessarily be identical, although the use of a single scale is considered (in his words) *ideal* and the latter case is presented in the examples.

2. Type II: diatonic root movement with symmetric chord structures[5]. Schillinger suggests using this type of progression to create higher tension and more harmonic variation than is possible within the Type I diatonic system.

3. Type III: symmetric chord structures and root movement[6].

Chord structures may be of either constant (e.g., S_9 only) or variable tension. Let us consider a first example, demonstrating the three forms of harmonic continuity for constant chord structure.

Example 1: Chord progression using constant tension hybrid 5-part harmony.
We will base the example on the following diatonic 7-pitch scale: $E_0 = \{p_{1,\ldots,7}\} = \{d - e - f\sharp - g - a - b\flat - c\sharp\}$. The chord structures are based on the first expansion (into thirds) of this scale $E_1 = \{d - f\sharp - a - c\sharp - e - g - b\flat\}$. The diatonic root progression is based on the following continuity

[3] Resolution of the dissonant 7th may be delayed until a few chords later, after a sequence of slurred notes.

[4] The characteristic preparation consists of introducing this pitch as a common tone in the preceding chord, where it is a consonant function, i.e., 1, 3, or 5. Next-best is an upward stepwise or leap movement into the dissonant 7th.

[5] Symmetric means that all chord structures are derived from the same diatonic scale and then transposed to the current root.

[6] Symmetric root movement means equal division of the octave into either two diminished 5ths, three major 3rds, four minor 3rds, six major 2nds or twelve minor 2nds.

$\vec{R} = R_3 + 2\{R_3 + R_7\} + 3R_3 + R_5$ (positive root cycles only). The root cycle, starting on the tonic pitch d is[7]:

$$\vec{R} = d \overset{R_3}{-} b\flat \overbrace{\overset{R_3}{-} g \overset{R_7}{-} a \overset{R_3}{-} f\sharp \overset{R_7}{-}}^{2\{R_3+R_7\}} g \overbrace{\overset{R_3}{-} e \overset{R_3}{-} c\sharp \overset{R_3}{-}}^{3R_3} a \overset{R_5}{-} d.$$

The constant chord structure we will use is S_9; on the root $R = d$ this yields $S_9 = \{d\}-\{f\sharp-a-c\sharp-e\}$, i.e., a minor 7th chord $F\sharp m_7$ over a root d, also written as $F\sharp m_7/D$.

The diatonic progression, shown in Fig. 1.a, demonstrates the resolution of the dissonant 7th in the upper chord structure by stepwise downward movement. The other voices move stepwise or are slurred to the next chord (note the enharmonic equivalence). There is a general downward trend[8], continuous close position voicing, and no crossing of voices. Occasional parallel perfect 5ths may occur in the inner voices. The 7-pitch scale and root progression were carefully constructed to obtain familiar extended chord structures on both the tonic d and dominant a.

The diatonic-symetric progression is shown in Fig. 1.b. The upper chord structure is based on the constant S_9 structure on the root d, i.e., $F\sharp m_7/D$, which is then appropriately transposed for each subsequent root. Now we begin to see upward stepwise movement in the inner voices; see e.g., the $f\sharp - f\natural - f\sharp$ movement in the lowest voice of the upper structure.

Symmetric progression is demonstrated in Fig. 1.c for various equal divisions of the octave, notated as $R(\sqrt[n]{2})$. (Equal division of the octave into chromatic steps, $R(\sqrt[12]{2})$, is not shown here). □

To construct similar examples for other 7-pitch diatonic scales and for other constant tension chord structures is left as an exercise to the reader.

Example 2: Chord progression using variable tension hybrid 5-part harmony.
We will construct a harmonic progression with variable tension using the continuity

$$\vec{S} = S_5 + S_9 + S_{13} + S_9 + S_7 + S_9 + S_5.$$

We will use the 7-pitch diatonic scale from the previous example, but now the root cycle sequence is

$$\vec{R} = R_7 + R_3 + R_5 + 2R_7 + R_5.$$

See Fig. 2.a, and again note the treatment of the dissonant 7th in the upper structure. There is either immediate resolution by stepwise downward movement, or a slurred note into the next chord (in case of a common tone). In the latter case, the requirement for downward resolution is dropped as can be seen in the figure, where the slurred g in the lower voice of the upper structure finally moves upward. This is allowed, since moving from the S_9 chord on the root $f\sharp$ to the S_7 chord on root g, the $f\sharp$ in the upper voice has become the dissonant note, that needs resolution (downward step into e).

The example also demonstrates the voice leading for a partially open voicing, as can be seen in Fig. 2.b and the second progression in Fig. 2.c. We still see the general downward tendency, the occasional parallel perfect 5ths, and the avoidance of voice crossing. □

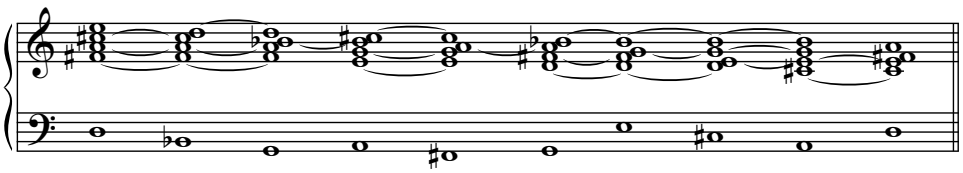
Play this example on a keyboard to experience the combination of varying chord tension and smooth voice leading during this continuity. The next example will demonstrate the combination of various aspects of hybrid 5-part harmony,

[7] Note the use of grouping symbols { }. Multiplicating a group by an integer a means that this group has to repeated a times. Therefore, $2\{R_3 + R_7\} = \{R_3 + R_7\} + \{R_3 + R_7\} = R_3 + R_7 + R_3 + R_7$, not $2R_3 + 2R_7 = R_3 + R_3 + R_7 + R_7$.

[8] Schillinger discusses two techniques for countering this downward trend, i.e., exchange and octave-inversion of pairs of common tones at the interval of a third.

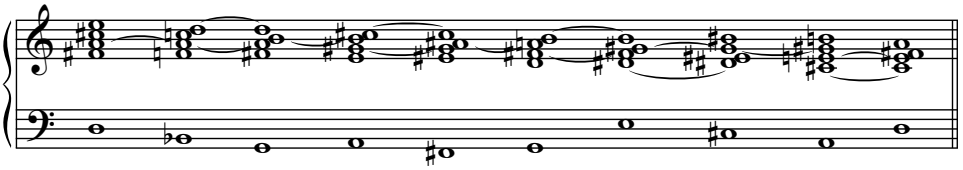
(a)

Type I: Diatonic harmonic progression (constant tension S_9)



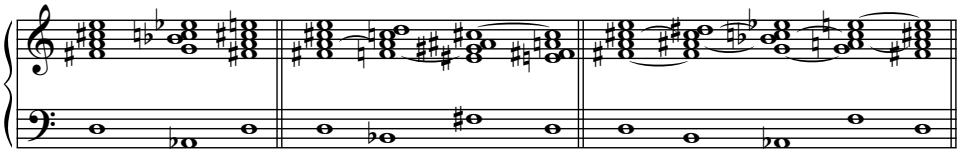
(b)

Type II: Diatonic-symmetric harmonic progression

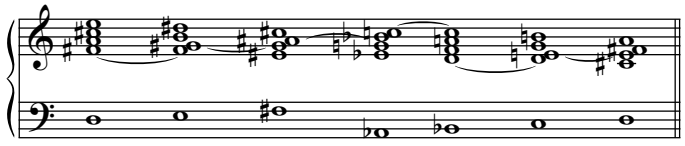


(c)

Type III: Symmetric harmonic progression



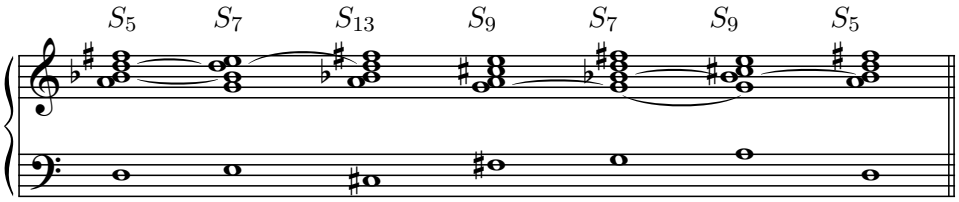
$R(\sqrt[2]{2})$ $R(\sqrt[3]{2})$ $R(\sqrt[4]{2})$



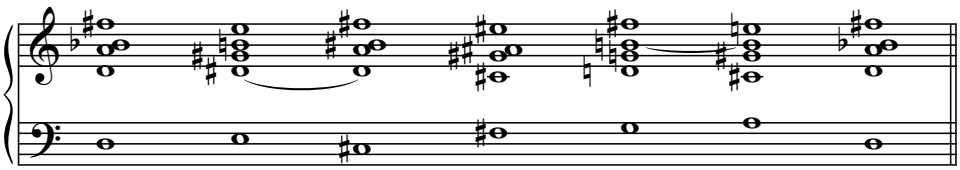
$R(\sqrt[6]{2})$

Figure 1: Hybrid 5-part harmonic continuity. *Constant tension* S_9 progression in the diatonic system (a), diatonic-symmetric system (b) and symmetric system (c). Note the stepwise downward movement of the dissonant 7th interval in the upper structure and the voice leading in the other upper voices (stepwise movement, no crossing voices, general downward tendency).

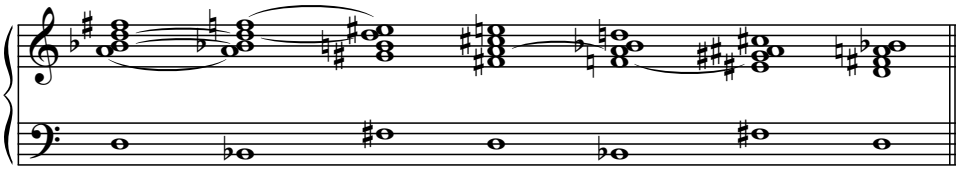
(a)
Type I: Diatonic harmonic progression (variable tension)



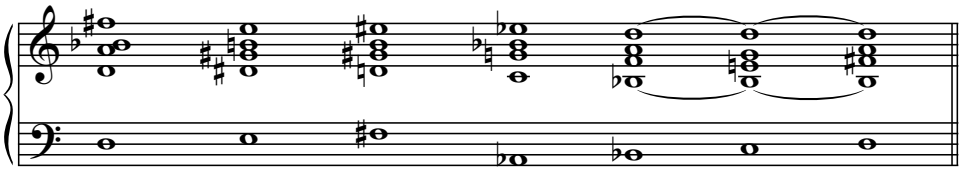
(b)
Type II: Diatonic-symmetric harmonic progression



(c)
Type III: Symmetric harmonic progression



$R(\sqrt[3]{2})$



$R(\sqrt[6]{2})$

Figure 2: Hybrid 5-part harmonic continuity. *Variable tension* progression in the diatonic system (a), diatonic-symmetric system (b) and symmetric system (c). Note the stepwise downward movement of the dissonant 7th interval in the upper structure and the voice leading in the other upper voices (stepwise movement, no crossing voices, general downward tendency).

Example 3: Hybrid 5-part harmonic progression with variable tension, progression type, root cycles and voicing.
We will construct a harmonic progression with variable tension using the chord continuity

$$\vec{S} = S_5 + S_9 + S_5 + S_{11} + 4S_9 + S_5 + S_{13} + S_7 + S_9 + S_5.$$

We will use the 7-pitch diatonic scale from the first example, but now the root cycle sequence is

$$\vec{R} = \overbrace{R_7 + R_5 + R_{-3} + R_5}^{\text{Diatonic}} + \overbrace{3R(\sqrt[4]{2})}^{\text{Symmetric}} + R_7 + \overbrace{R_{-3} + R_5 + R_7}^{\text{Diatonic-Symmetric}} + R_5,$$

where after the 4th chord $S[4]$ there is a change from a variable tension Type I Diatonic progression to a constant tension (S_9) Type III Symmetric progression. The example returns to variable tension diatonic progression after $S[8]$, then followed by 3 chords, $S[10]$ to $S[12]$, in the Type II Diatonic-Symmetric system, before the diatonic closing cadence $S[12] \rightarrow S[13]$.

A possible voicing of this progression is shown in Fig. 3. Dashed barlines indicate the change in chord progression type. Check the resolution of the dissonant 7th in the upper structure. Occasionally there are exact parallel 5ths in the inner voices of the upper structure (see $S[3] \rightarrow S[4]$ and $S[12] \rightarrow S[13]$).

There is twice the occurrence of a negative root cycle (from $S[3] \rightarrow S[4]$ and $S[9] \rightarrow S[10]$). These have received special handling with respect to the subsequent root progression (not from the point of view of voice leading; note how the dissonant 7th in the upper structure is still properly resolved by downward stepwise movement). In both cases the subsequent root progression is based on the positive root cycle and the combination of both root movements leads to an overall positive root movement. This can be notated as:

$$\vec{R} = \overbrace{R_{-3} + R_5}^{R_3},$$

or

$$\vec{S} = \overbrace{S[3] \overset{R_{-3}}{-} S[4] \overset{R_5}{-} S[5]}^{R_3}, \quad \vec{S} = \overbrace{S[9] \overset{R_{-3}}{-} S[10] \overset{R_5}{-} S[11]}^{R_3}.$$

Inspect the actual roots to confirm this root cycle property; for $S[4] \rightarrow S[6]$ we have $a - c\sharp - (g\flat = f\sharp)$, and for $S[9] \rightarrow S[11]$ we have $b\flat - d - g$.

The example demonstrates another feature. In $S[1] \rightarrow S[2]$ and $S[9] \rightarrow S[10]$ there is the swapping of common tones ($b\flat \rightleftharpoons d$ and later $b\flat \rightleftharpoons g$, both neighbouring pitches in close voicing); this technique can be used to counter the general downward trend in all voices.

This example has been carefully constructed to show the combination of various aspects. Play it on the keyboard and experience the high tension, quite dissonant, atmosphere; this is caused by the high density of effects over a 13-chord sequence. A more relaxed distribution of effects over a more extended chord sequence might be preferred for a more musical effect. □

This concludes our discussion of the hybrid 5-part technique. In the Schillinger book the 5-part technique is presented at the the end of the chapter on the S_7 chord. The following section will discuss the hybrid 4-part technique, which differs quite a bit from the 5-part equivalent; in Schillinger's book it is discussed at the end of the chapter on the S_{11} chord structure.

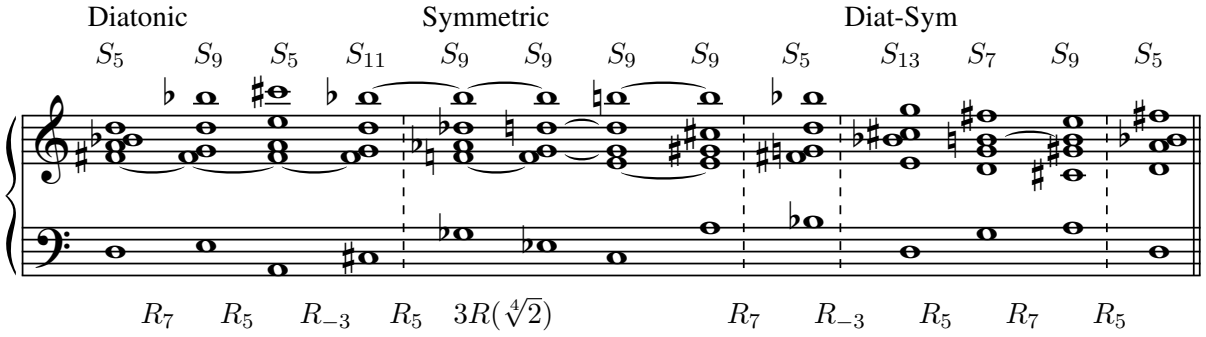


Figure 3: Hybrid 5-part harmonic continuity with variable tension, progression type, root cycles and voicing. Note the swapping of common tones to counter the downward trend (see $S[1] \rightarrow S[2]$ and $S[9] \rightarrow S[10]$).

Table 2: Hybrid 4-part chord structures (those marked with an asterisk are less commonly used)

Upper	5	5	7	7	9	9	11	13	13
Structure	3	3	5	3	7	7	9	9	11
	1	13	3	1	3	1	7	7	7
Root	1	1	1	1	1	1	1	1	1
Chord structure	S_5	S_5^*	S_7	S_7^*	S_9	S_9^*	S_{11}	S_{13}	S_{13}^*

4 Hybrid 4-part harmony

4.1 Chord structure

Valid hybrid 4-part chord structures are shown in Table 2. The upper structure now contains 3 pitches, $p[1], p[2], p[3]$. Three upper structures are regular triads S_5 (see the 1st, 3rd and 7th column), two upper structures contain the three essential functions of a 7th chord structure (root, third and seventh, $j = 1,\ 3,\ 7$, see the 4th and 8th column).

4.2 Upper structure voice leading

In the hybrid 4-part technique Schillinger allows greater freedom in voice leading; upper structure voice leading may be based on any of the 6 fundamental *transformations* possible for 3-pitch chord structures (see the Appendix). Given this set of possible transformations, Schillinger proposes the following guidelines:

1. When two neighbouring chords have *identical* chord structure[9], do not use the $S[j] \xrightarrow{\overline{123}} S[j+1]$ (all pitches constant) transformation, since this will lead to complete parallelism.

2. When a group of two subsequent chords is *partly identical*, use a constant single pitch transformation ($S[j] \xrightarrow{\bar{1}} S[j+1]$, $S[j] \xrightarrow{\bar{2}} S[j+1]$, or $S[j] \xrightarrow{\bar{3}} S[j+1]$), when this leads to a parallel

[9]Identical chord structure means that both chords are constructed from the same column in Table 2, and therefore contain the same chord functions.

desirable interval with the bass (parallel third or sixth). Do not use these partly parallel transformations, when they yield a consecutive parallel seventh or ninth with the bass.

3. When two subsequent chords are totally different (i.e., no equal chord function in the upper structure. Examples are the combination of 1st and 8th, or 2nd and 9th column in Tab. 2), then the transformation that keeps all pitches constant, $S[j] \xrightarrow{\overline{123}} S[j+1]$, is the most favourable. Schillinger summarizes these guidelines as: *homogeneous* chord structures are compensated by *heterogeneous* transformations (i.e., non-constant functions, clockwise or counterclockwise rotation of chord functions), and the reverse. The main goal is to prevent the occurrence of parallel octaves and fifths in the voice leading.

This upper structure voice leading guidelines say nothing about preparation or proper resolution of the higher chord functions $j = 7,\ 9,\ 11$ or 13; in that respect there is more freedom than in the hybrid 5-part harmonic technique. Some of the chord transformations may lead to crossing voices, although in practice (see the examples) this happens rarely.

4.3 Root movement

Root movement is determined by positive root cycles only, i.e., $R_3,\ R_5$ or R_7.

When creating an overview of all possible voice leadings for a group of two hybrid 4-part harmony we must remember that there are five chord types ($S_5,\ S_7,\ S_9,\ S_{11}$ and S_{13}), six transformations and three root cycles; this leads to a huge set of combinations, with a total n_t of

$$n_t = \overbrace{2(5 \times 4 \times 3 \times 2)}^{\text{chord structure type}} \times 6 \times 3 = 4320$$

possibilities.

Schillinger's book does not present all combinations as score examples, and most are left to the student. Here we will show the voice leading options for one identical and one non-identical group, and discuss the result. We will base the diagrams on the familiar diatonic 7-pitch scale $E_0 = \{p_{1,\ldots,7}\} = \{d - e - f\sharp - g - a - b\flat - c\sharp\}$. The result is shown in Fig. 4 and 5, where also crossing voices and parallelism are indicated.

The clockwise and counterclockwise transformations maintain the close voicing, the three constant single pitch transformations change the voicing type from close to open voicing, while the triple constant pitch transformation obviously leads to complete parallelism in the upper structure.

Parallelism between upper voices and bass will also occur; sometimes the parallel intervals are favourable (see the thirds in Fig. 4), sometimes these are unfavourable (see the fifths in Fig. 4, or the parallel seventh in the last transformation in Fig. 4.a).

Remember Schillinger's statement about homogeneous structures in combination with heterogeneous transformations (and the reverse), and see whether the figures provide sufficient evidence to confirm that fact. In that case Fig. 4 should have less objectionable features on the left, while Fig. 5 should yield better voice leading on the right for constant pitch transformations. The evidence is not convincing (author's opinion), but the second set of diagrams is a somewhat extreme progression ($S_{11} - S_5$) and we would really have to see the properties for a multitude of possibilities.

4.4 Application and examples

After demonstrating the options for voice leading when moving from one hybrid 4-part chord structure to the next, we will now discuss a number of examples for harmonic continuities.

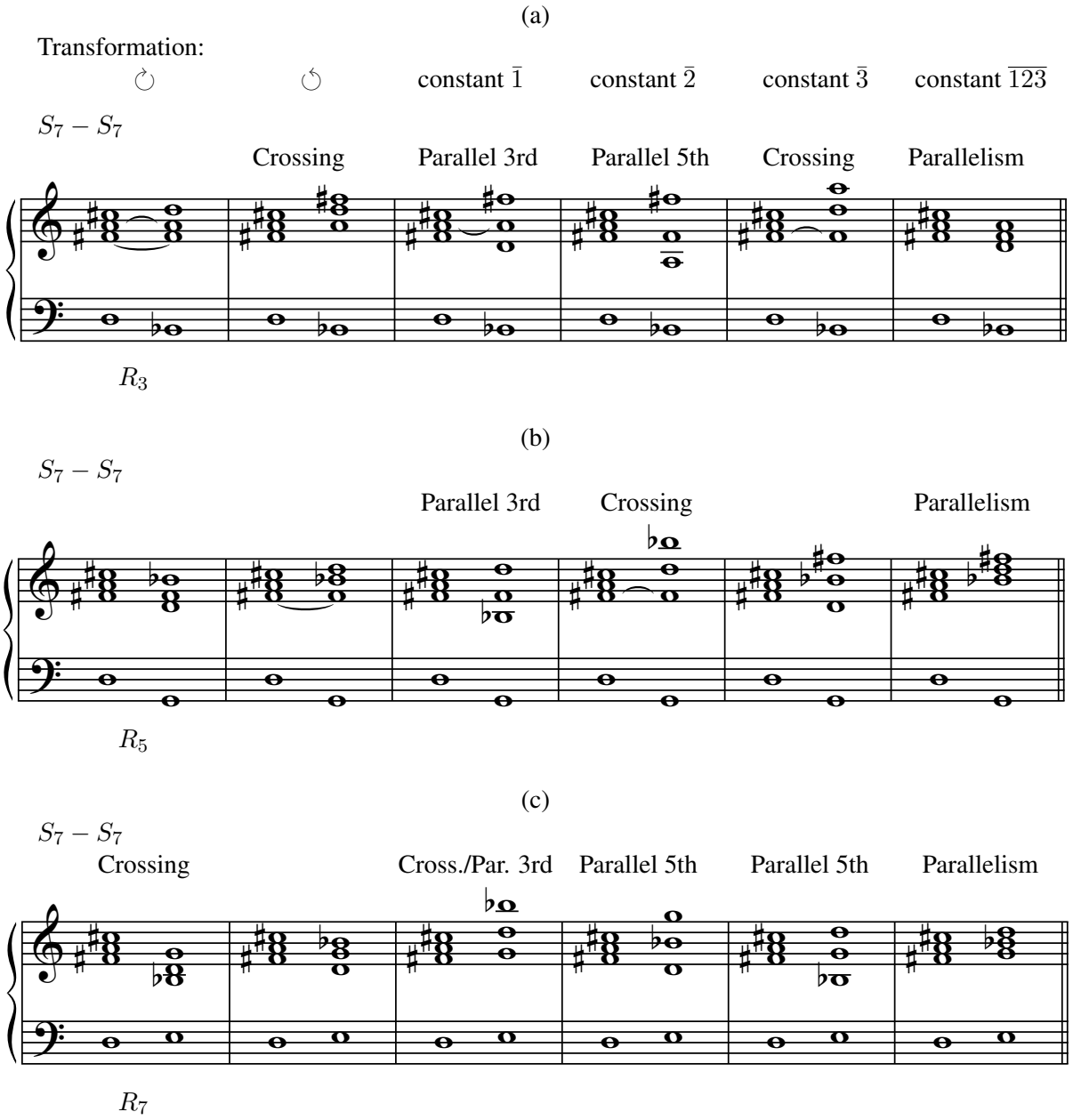


Figure 4: Hybrid 4-part transformations. *Constant tension* S_7 for root cycle R_3 (a), R_5 (b) and R_7 (c). All six possible transformations are presented (see marks at the top of the example). Parallel movement is considered internally in the upper structure or with relation to the bass.

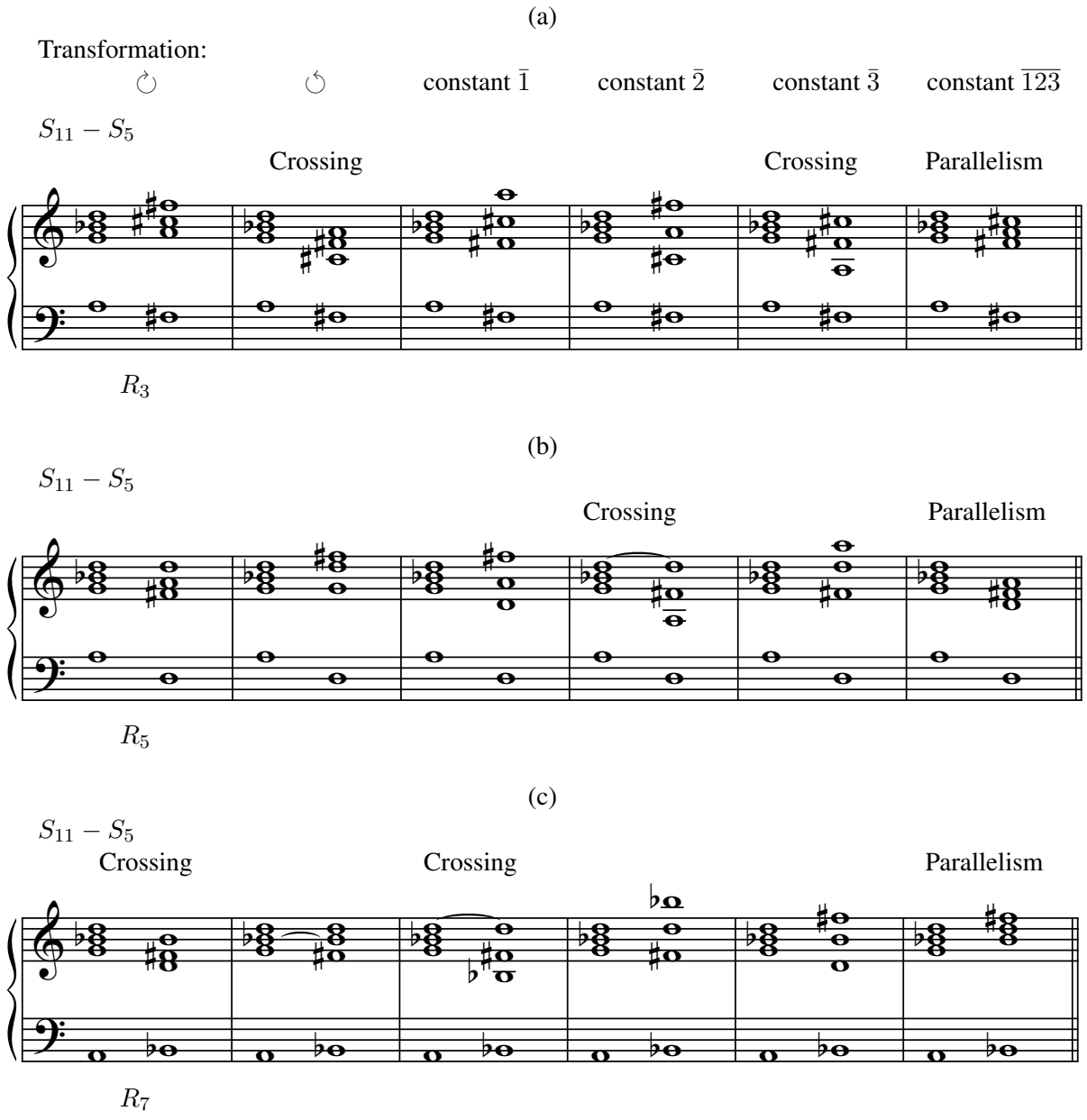


Figure 5: Hybrid 4-part transformations. *Variable tension* $S_{11} - S_5$ for root cycle R_3 (a), R_5 (b) and R_7 (c). All six possible transformations are presented (see marks at the top of the example). Parallel movement is considered internally in the upper structure or with relation to the bass.

Example 4: Hybrid 4-part harmonic continuity with constant tension for all progression types. Once again, we start from the diatonic 7-pitch scale: $E_0 = \{p_{1,\ldots,7}\} = \{d - e - f\sharp - g - a - b\flat - c\sharp\}$. However, now we will use a different *mode* of that scale, starting on the tonic e. We will use a constant tension S_9 chord structure, that contains the functions $S_9 = \{1\} - \{3 - 7 - 9\}$ (see Table 2). On the tonic root this yields the following chord: $e - g - d - f\sharp$, in conventional chord notation $Em^9_{\text{no }5}$. The diatonic root progression is given by $\vec{R} = R_5 + 2\{R_7 + R_3\} + R_3 + R_7$ (positive root cycles only).

The voice leading for this harmonic continuity is shown in Fig. 6. The transformations in all progression types aim at smooth voice leading, i.e., slurring common notes and minimize the movement in each voice (stepwise movement preferred). Occasionaly this leads to crossing voices in the upper structure (indicated in the score, see Fig. 6.a and b). Such crossing can be prevented by a very wide voicing and leapwise movement, as is demonstrated in the first symmetric root progression in Fig. 6.c.

Note the general downward trend in the voice leading in the Type I, II and Type III for $R(\sqrt[6]{2})$. Also note that the highest function in the upper structure (i.e., the 9, which creates a dissonant 7th interval with the 3) frequently is properly prepared. This means that it is either being slurred from the preceding chord or it is approached by an upward step. The dissonant interval is also frequently resolved by downward stepwise movement of the upper function. Doing so, the voice leading adheres to the principles of traditional tonal harmony. An exception is shown in the first two chords in Fig. 6.a, where the $b\flat$ is approached by a downward leap from d.

The transformations for the smooth voice leading attempt in the Type III progression are indicated in the score. These were not determined a priori; they turn out to be sequences of either clockwise or counter-clockwise circular permutations of chord functions.

Since we use constant tension S_9 chord structures, there is the danger of undesirable parallel 7th or 9th intervals between upper structure functions and the root. The voice leading shows that these have been prevented. □

Next, we will present a somewhat more extended example with variable chord tension.

Example 5: Hybrid 4-part harmonic continuity with variable tension and mixed chord progression type.
We will construct a harmonic progression with variable tension using the chord continuity

$$\vec{S} = S_9 + 3S_5 + 2\{S_9 + S_{11}\} + S_7 + 3S_{13} + S_{11} + S_9.$$

There is a deliberate gradual build-up of chord tension towards the end of the progression, with a return to a more stable S_9 on the final chord. We will use the 7-pitch diatonic scale (on tonic e) from the previous example, but now the root cycle sequence is

$$\vec{R} = \overbrace{R_3 + R_7 + R_5 + R_7}^{\text{Diatonic}} + \overbrace{3R(\sqrt[4]{2})}^{\text{Symmetric}} + R_7 + R_3 + \overbrace{R_3 + R_7}^{\text{Diat.-Symm.}} + R_5 + R_7,$$

where after the 4th chord $S[4]$ there is a change from Type I Diatonic progression to Type III Symmetric progression. The example returns to variable tension diatonic progression after $S[8]$, then followed by 3 chords, $S[10]$ to $S[12]$, in the Type II Diatonic-Symmetric system, before a diatonic closing cadence $S[12] \rightarrow S[13] \rightarrow S[14]$. The root progression contains positive root cycles only and has been carefully constructed to start and return on the tonic root of this modal scale.

The transformations (not indicated in the score in Fig. 7) have been chosen to yield smooth voice leading (slurred common notes, stepwise movement preferred, maximizing proper preparation and downward stepwise resolution of dissonant upper functions). An occasional improper resolution of the upper

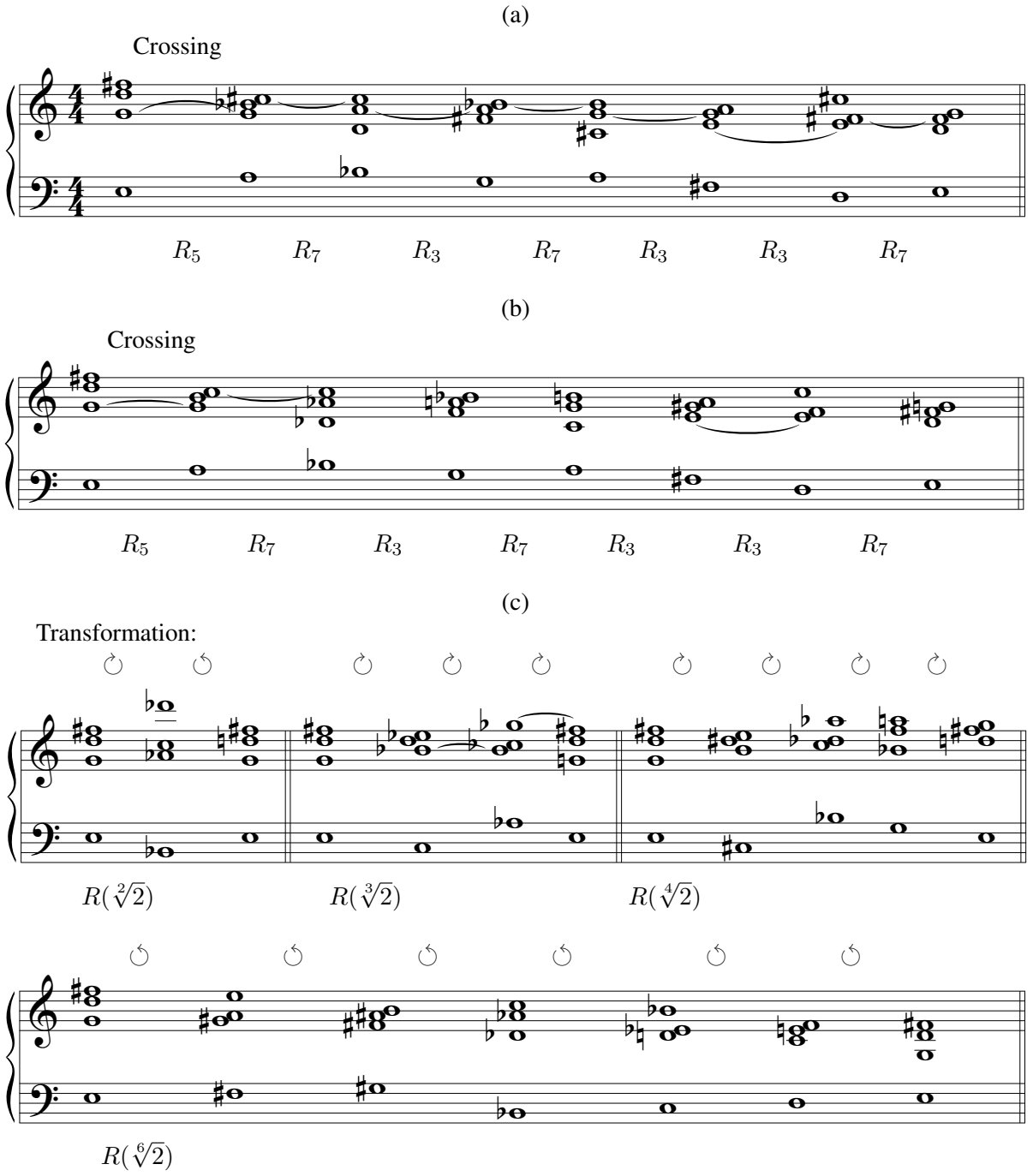


Figure 6: Hybrid 4-part harmonic continuity. *Constant tension* S_9, for Type I diatonic (a), Type II diatonic-symmetric (b), and Type III symmetric (c) harmonic progression.

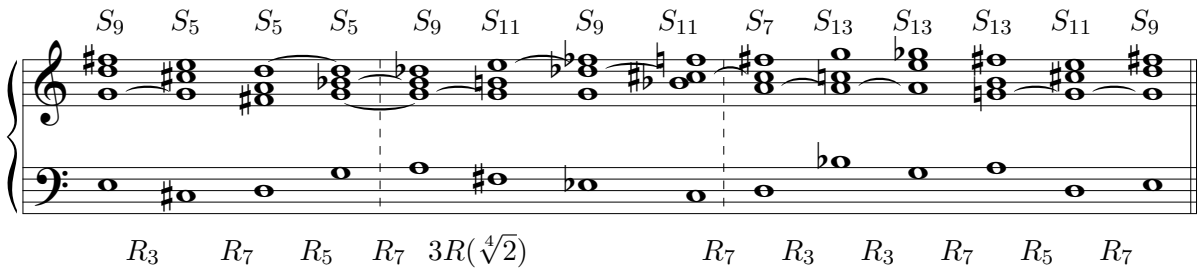


Figure 7: Hybrid 4-part harmonic continuity. *Variable tension*, for mixed type harmonic progression.

structure function 9 is shown in the progression $S[7] \rightarrow S[8]$, with the upward stepwise movement $f\flat \nearrow f\natural$. □

Let us have a look at another variable tension example.

Example 6: Hybrid 4-part harmonic continuity with variable tension.
In this example we will use two modal variants from the diatonic scale, the first on the tonic root g, the second on e. The variable tension chord structures are indicated in the score (see Fig. 8). The first modal variant is based on a Type I Diatonic progression, the second on a Type III Symmetric progression.

Again, we aim for a fairly smooth voice leading. Both examples are free of vocie crossing, nor do they contain parallel 7th or 9th movement between upper functions and roots. Dissonant intervals in the upper structure are regularly resolved properly. Note the exception in Fig. 6.a in $S[2] \rightarrow S[3]$ with the upward leap of the dissonant major 7th $f\sharp \nearrow a$, or at the end $S[7] \rightarrow S[8]$ with the upward stepwise movement $d\flat \nearrow d\natural$.

The second modal variant on e deliberately alternates $S_7 \rightleftharpoons S_{11}$; these chord structures have only one upper structure common function, i.e., 7. Therefore, this heterogeneous group should yield a preferrence for constant function transformations, in case of smooth voice leading. The marks above the staff indicate that there is indeed some evidence to support this rule (see the constant $\overline{123}$ transformations). However, slurring common notes leads to a circular permutation type of transformation.

Note the tonic root chord, a half-diminished 7th chord $E_\emptyset$, that opens the example. Fortunately the closing chord on the same root has a different structure (i.e., tension S_{11}), since the tonic triad S_5 is highly unsuitable for acting as a stable structure that will confirm the tonality of the example[10]. □

The final example includes a few additional aspects.

Example 7: Hybrid 4-part harmonic continuity with fully heterogeneous variable tension chord structures.
In this example we will investigate the effect of transformation types on the voice leading, when dealing with completely heterogenous chord structures. Therefore, the harmonic continuity is based on alternating $S_5 \rightleftharpoons S_{11}$ chord structures Remember, that in hybrid 4-part chord structures the upper structure of the S_5 tension chord contains the functions $\{1-3-5\}$, whereas the S_{11} upper structure consists of $\{7-9-11\}$. So, indeed, these structures are completely dissimilar (no overlapping functions, although both are perfect triads).

The root sequence is given by

$$\vec{R} = 2R_3 + 2R_5 + 2R_7 + \overbrace{R_{-3} + R_5}^{R_3} + R_3,$$

[10]This example is of academic nature. In case of writing real music, I would have adapted the structures to yield a more stable chord on the tonic root.

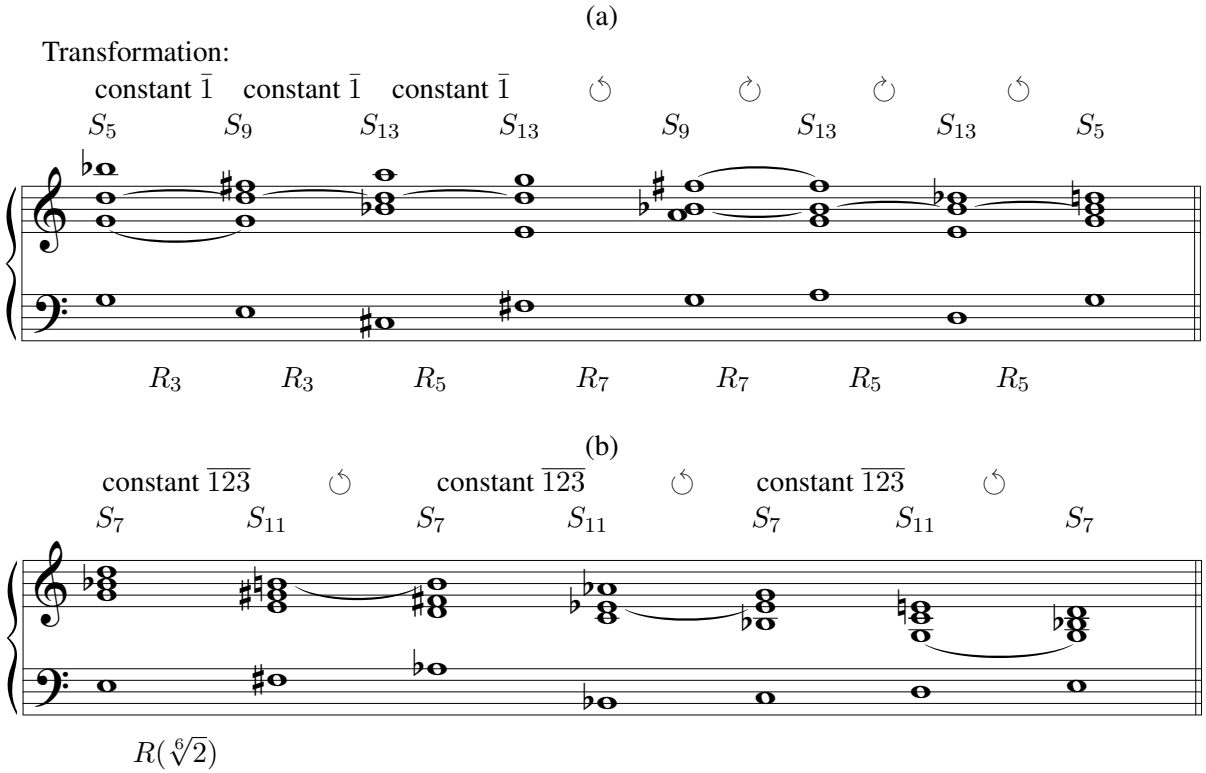


Figure 8: Hybrid 4-part harmonic continuity. *Variable tension*, for Type I diatonic (a), and Type III symmetric (b) harmonic progression.

a highly artificial sequence, since it deliberately contains repeated positive root cycles (e.g., $2R_3$), but also a composite group of a *negative* plus a positive root cycle (the $R_{-3} + R_5$ group).

The first solution (see Fig. 9.a) is based on variable transformations, aiming for smooth voice leading. The resulting transformation type is either clockwise or anti-clockwise circular permutation of chord functions. It is not a constant function type of transformation, the guideline given by Schillinger for handling of heterogeneous chord groups. The constant $\overline{123}$ transformation (not presented here) would lead to a set of parallel triads in the upper structure with unacceptable parallel voicing.

Applying (or better, forcing) a constant function transformation is demonstrated in Fig. 9.b and c. Note the alternating close and open type voicing in the upper structure, and the occasional crossing voices. There are many wide leaps in the upper voices; using this type of transformation (as opposed to the smooth voice leading solution in Fig. 9.a) would affect the orchestration in the case of writing real music. I would apply the variable transformation solution to a harmonic (long notes, chorale type) setting for bowed strings (or for trombones at the octave below), whereas the constant pitch solution with its wide leaps suggests pizzicato strings or woodwind accents. □

The final example contained a *negative root cycle* in a hybrid 4-part harmonic continuity. This does not appear in the Schillinger book, and therefore was not presented as a root movement option at the beginning of this section (there, as a rule, we allowed positive root cycles only). However, voice leading diagrams like Fig. 4 and 5 may also be constructed for all six transformations and for all three negative root cycles (this is left as an exercise to the student). Whether this additional destabilising effect to tonality in the case of hybrid 4-part higher tension chords is acceptable musically, is a matter of taste. The last example was meant to make the reader conscious of that optional extension to the technique.

5 Conclusion

This document presents two extensions to the traditional theory of diatonic harmony, as proposed by Schillinger [1], i.e., hybrid 5- and 4-part harmony. Rules and guidelines for these techniques were followed by detailed discussion of examples.

Note, however, that only the chord progression has been considered. The examples shown herein do not yield musical beauty; they only demonstrate voice leading aspects, when using the hybrid structures in a harmonic continuity. In order to create real music other elements such as melody and rhythm must be included in the score.

Here we have discovered a technique to deliberately create varying tension over a chord progression. The composer may use this toolbox of techniques, derived from traditional harmony, and play with and control chord tension, chord progression types and transformations; this may lead to useful and interesting musical results. Try it!

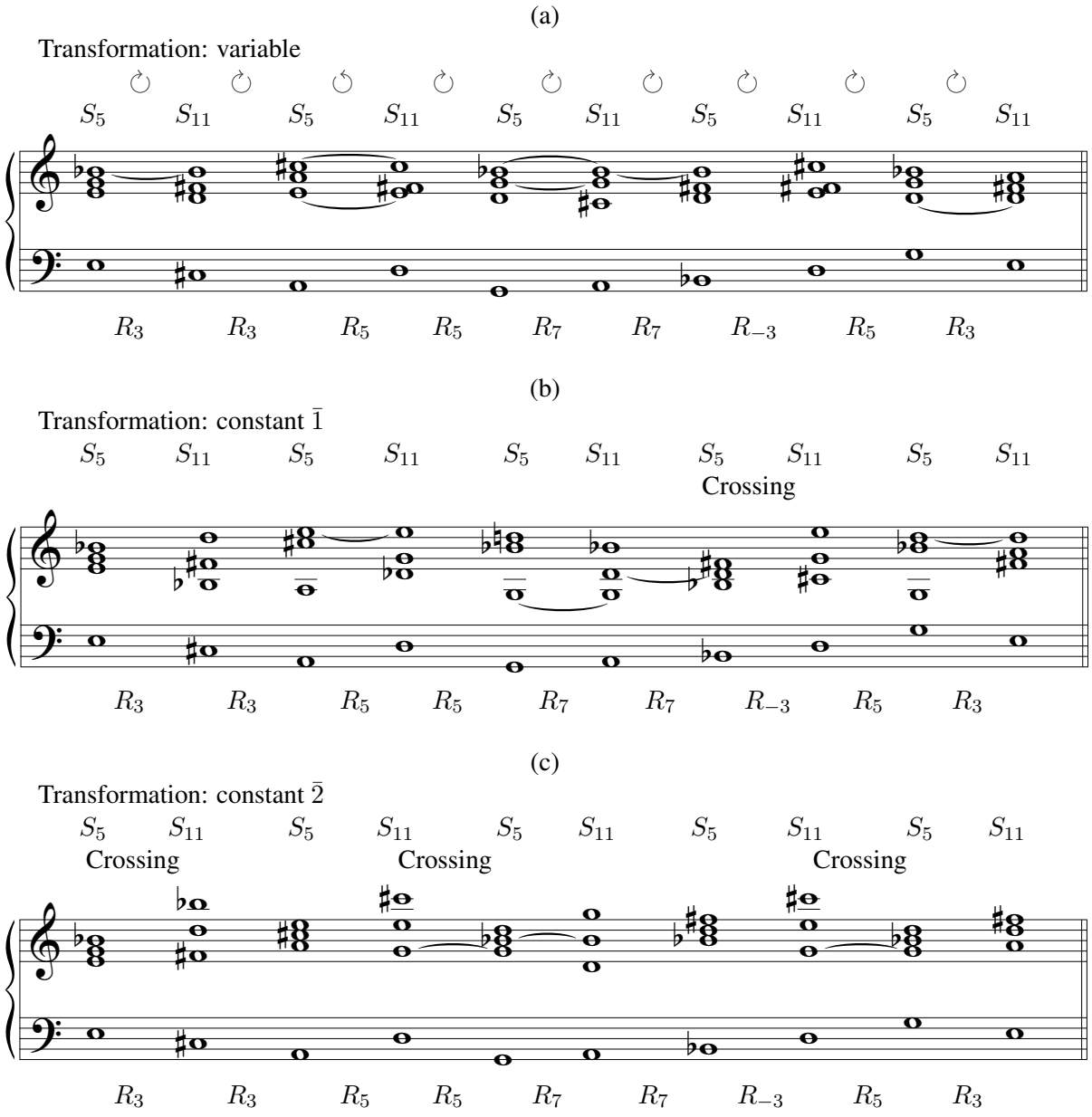


Figure 9: Hybrid 4-part transformations. *Variable tension* $S_5 \rightleftharpoons S_{11}$, Type I diatonic harmonic continuity, for variable (a), constant $\bar{1}$ (b), and constant $\bar{2}$ (c) transformation. Note the negative root cycle R_{-3} in the harmonic progression. Also note the alternation between closed and open voicing, and the voice crossing for the constant $\bar{1}$ and $\bar{2}$ transformations.

Appendix: Transformation of chordal functions

We will consider chord structures S with 3 pitches or chord functions $p[i], i = 1, \ldots, 3$ and will present the *voice leading* options during the *transformation* from one chord $S[j]$[11] to the next $S[j+1]$. We will designate the i-th function in the j-th chord as $p[i,j]$. We therefore can write the triadic chord structure as:

$$S[j] = \begin{pmatrix} p[3,j] \\ p[2,j] \\ p[1,j] \end{pmatrix}.$$

Since the chord structure contains 3 pitches this yields $3! = 1 \times 2 \times 3 = 6$ optional transformations:

1. A clockwise circular permutation of chord functions $S[j] \circlearrowright S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[1,j+1], \\ p[2,j] &\rightarrow p[3,j+1], \\ p[1,j] &\rightarrow p[2,j+1]. \end{aligned}$$
2. A counterclockwise circular permutation of chord functions $S[j] \circlearrowleft S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[2,j+1], \\ p[2,j] &\rightarrow p[1,j+1], \\ p[1,j] &\rightarrow p[3,j+1]. \end{aligned}$$
3. Keep the 1st chord function constant, while swapping the 2nd and 3rd function. This is notated as $S[j] \xrightarrow{\bar{1}} S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[2,j+1], \\ p[2,j] &\rightarrow p[3,j+1], \\ p[1,j] &\rightarrow p[1,j+1]. \end{aligned}$$
4. Keep the 2nd chord function constant, while swapping the 1st and 3rd function. This is notated as $S[j] \xrightarrow{\bar{2}} S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[1,j+1], \\ p[2,j] &\rightarrow p[2,j+1], \\ p[1,j] &\rightarrow p[3,j+1]. \end{aligned}$$
5. Keep the 3rd chord function constant, while swapping the 1st and 2nd function. This is notated as $S[j] \xrightarrow{\bar{3}} S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[3,j+1], \\ p[2,j] &\rightarrow p[1,j+1], \\ p[1,j] &\rightarrow p[2,j+1]. \end{aligned}$$
6. Keep all chord function constant. This is notated as $S[j] \xrightarrow{\overline{123}} S[j+1]$, where
$$\begin{aligned} p[3,j] &\rightarrow p[3,j+1], \\ p[2,j] &\rightarrow p[2,j+1], \\ p[1,j] &\rightarrow p[1,j+1]. \end{aligned}$$

In this document transformations types 1 and 2, i.e. (counter)clockwise circular permutation of chord functions, are referred to as *heterogeneous* transformations (using Schillinger parlance).

[11] Note that we use integer subscripts (e.g., S_7) to indicate the *tension* of the chord, while the integer index $[j]$ indicates the position of the chord in a sequence (i.e., in the time domain). The integer index $[i]$ indicates the simultaneous sounding of pitches $p[i]$ in a chord structure (ordered from low to high pitches).

References

[1] Joseph Schillinger. *The Schillinger System of Musical Composition*, volume I and II of *Da Capo Press Music Reprint Series*. Da Capo Press, New York, fourth edition, 1946. ISBN 0-306-77521-2 and 0-306-77522-0. xxiii + 1640 pp.

