Guide to the Theory of Rhythm

Schillinger System of Musical Composition

Version 1.2

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Preface

This booklet is a guide to help you understand Book I *The Theory of Rhythm* from *The Schillinger System of Musical Composition* [3]. Each chapter from the original book will be commented upon and examples will be presented and discussed.

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Chapter 1

Notation system

Keywords: notation, definition.

Chapter 1, [3] p. 1, introduces alternative systems of rhythmic pattern notation: numbers, graphs and musical notes. In the book the usefulness of graphs is stressed (obviously, because of the visualisation aspect), but here we will limit ourselves to the numbers and musical notation system.

> Comment:
> The Theory of Rhythm deals with temporal aspects of music. In that field there are not many textbooks; there are many more books about harmonic structures and chord progressions available. However, here's a number of suggestions for books that discuss rhythm and temporal aspects of music: see [1, 2]

1.1 Graphing Music

In the book the analogy between acoustic waveforms (periodic patterns, displaying the sound amplitude vs. time) and durations (stressed accents) is used to introduce the square wave graphical notation of musical attacks.

Here we will use the analogy of *clocks* or *metronomes* ticking (short pulses of sound) at regular intervals. The *time instants* of the ticking will be represented as numbers and most of the techniques (the mathematical processes, the arithmetic) will be done in numbers (don't be afraid, this is all very simple and the analogy of the ticking metronomes will help to understand the results).

1.2 Forms of Periodicity

Uniform periodicity can be achieved with a single metronome generating pulses at a constant rate; this is called *monomial periodicity*. The pulses are generated at discrete time intervals Δt, i.e., the smallest rhythmical time unit. Attacks will occur at multiples of this smallest time unit. We can write the time instant of the i-th attack t_i (from a total series of N attacks or pulses) in mathematical form as follows

$$t_i = (i-1)\Delta t, \quad i = 1, 2, \ldots, N. \tag{1.1}$$

Note that the first attack occurs at $t = 0$ (and not $t = 1\,\Delta t$); that seems a bit odd, but later we will see that this makes the arithmetic a lot easier to understand. We can represent the whole series of ticks as a *vector* $\vec{t}$ (note the small arrow over the symbol t) and therefore we may write

$$\vec{t} = [0\ 1\ 2\ \ldots\ (N-1)] \cdot \Delta t. \tag{1.2}$$

Let us look at an example.

△ **Example 1.1.** Monomial periodicity: attack series

▷ Consider metronome A ticking 5 times at intervals of 1 time unit (e.g., 1 second intervals). Then we have $N = 5$ and $\Delta t = 1$ and the series of attacks is written as

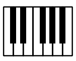
$$\vec{t}_A = [0\ 1\ 2\ 3\ 4].$$

▷ Another metronome B, generating 11 ($N = 11$) pulses at 3 time unit intervals ($\Delta t = 3$) will yield an attack series

$$\vec{t}_B = [0\ 1\ 2\ \ldots\ 11] \cdot 3 = [0\ 3\ 6\ \ldots\ 30].$$

▷ As a last example we will consider metronome C generating N pulses at n time unit intervals. In that case the attack series is

$$\vec{t}_C = [0\ 1\ 2\ \ldots\ N-1] \cdot n = [0\ n\ 2n\ \ldots\ (N-1)n].$$



Note that we have only indicated the time instant of a musical event, i.e., an attack (e.g., the staccato tones from a xylophone). The notation in the book uses $+$-signs, because the series there also indicate the *duration* of the attacks. As an analogy, consider an electronic keyboard, where you would press a specific key at the abovementioned time instant and keep the key depressed, until the next time event occurs. The result obviously is a series of repeated pitches with a specific duration. If the duration series is meant we will use the series with the $+$-signs between the terms.

△ **Example 1.2.** Monomial periodicity: duration series

▷ Returning to our previous example we would get for metronomes A to C the following attack duration series:

$$\begin{aligned}
a_A &= \sum_1^{N_1} \Delta t_1 = \sum_1^{5} 1 = 1+1+1+1+1, \\
a_B &= \sum_1^{N_2} \Delta t_2 = \sum_1^{11} 3 = 3+3+3+3+3+3+3+3+3+3+3, \\
a_C &= \sum_1^{N_3} \Delta t_3 = \sum_1^{N} n = n+n+n+\cdots+n.
\end{aligned}$$

Chapter 2

Interference of periodicities

Keywords: basic technique, interference, number theory.

This chapter introduces a basic technique for generating attack series. It is based in the interference pattern that results when two clocks or metronomes tick at a different time interval. These attack series may then be grouped using different numbers of time units per measure.

2.1 Binary synchronisation

Suppose there are two clocks or metronomes A and B ticking at different regular time intervals $\Delta t_A =$ 2 
$a\Delta t$ and $\Delta t_B = b\Delta t$, where a and b are integer numbers and Δt is the musical time unit (e.g., a quarter note or an eight note).

We assume that metronome B is ticking faster than metronome A, and therefore $\Delta t_B < \Delta t_A$. Metronome B is called the *minor generator*, metronome A is called the *major generator*.

Starting these two clocks at the same time instant will yield an attack pattern, called *the resultant*. The process of combining the two metronomes is called *interference* and since there are two clocks we call this *binary synchronisation*.

When metronome A and B produce an attack series, the interference process is notated as $a \div b$ and the resultant attack series is written as $r_{a \div b}$.

We determine the resultant time series by finding the combination (in mathematical terms, the *union*) of the two attack series $\vec{t}_A$ and $\vec{t}_B$, i.e.,

$$\vec{t}_r = \vec{t}_A \cup \vec{t}_B. \tag{2.1}$$

Given the attack series $\vec{t}_r$ we determine the duration series r by listing the difference between two consecutive terms in the attack series

$$r_i = t_{r,i+1} - t_{r,i}. \tag{2.2}$$

2.1.1 Uniform binary synchronisation

In the case of *uniform binary synchronisation* the faster metronome ticks at intervals of $\Delta t_B = 1$ time unit. Remember, the time unit my be any musical note duration, eg., $\Delta t_B = 1/4$ note or $\Delta t_B = 1/8$ note.

In this special case the time attack patterns will repeat after Δt_A time units. The book discusses the three most common cases of this interference $a \div b$.

Case 1: 2 and 1

Now $\Delta t_A = 2$ and $\Delta t_B = 1$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0], \quad \vec{t}_B = [0\ 1],$$

repeating itself after 2 time units. We determine the resultant from the combination of these two attack series and get the following attack and duration series

$$\vec{t}_r = [0\ 1], \quad r = \hat{1} + 1,$$

where $\hat{1}$ indicates an accented attack occurring every Δt_A time units.

Case 2: 3 and 1

Now $\Delta t_A = 3$ and $\Delta t_B = 1$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0], \quad \vec{t}_B = [0\ 1\ 2],$$

repeating itself after 3 time units. We determine the resultant from the combination of these two attack series and get the following attack and duration series

$$\vec{t}_r = [0\ 1\ 2], \quad r = \hat{1} + 1 + 1.$$

Case 3: 4 and 1

Finally, $\Delta t_A = 4$ and $\Delta t_B = 1$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0], \quad \vec{t}_B = [0\ 1\ 2\ 3],$$

repeating itself after 4 time units. We determine the resultant from the combination of these two attack series and get the following attack and duration series

$$\vec{t}_r = [0\ 1\ 2\ 3], \quad r = \hat{1} + 1 + 1 + 1.$$

> Comment:
> The process of uniform binary synchronisation leads to trivial rhythmic results: a series of attacks with equal duration where at regular intervals the attack receives an accent. More interesting resultants are obtained in the next section.

2.1.2 Non-uniform binary synchronisation

In the case of *non-uniform binary synchronisation* we have two metronomes A and B with $\Delta t_A > \Delta t_B$ (both an integer number) and $\Delta t_B \neq 1$. In that case the resultant attack series is repeated after $\Delta t_r = \Delta t_A \Delta t_B$ time units and the duration series will contain non-equal durations.

On [3] p. 14 we see the list of practical combinations of generators, here reproduced (somewhat re-arranged and complemented) as Table 2.1. We note that only the lower left triangular region of the table is filled; this is due to the fact that we must have $\Delta t_A > \Delta t_B$. Then there is a number of terms between brackets, e.g., $(2 \div 1)$; this is caused that in the case of the two generators having a common denominator, the interference group may be reduced to a simpler form. For example for $\Delta t_A = 4, (a = 4)$ and $\Delta t_B = 2, (b = 2)$ we may write

$$a \div b = 4 \div 2 = (2 \cdot 2) \div (1 \cdot 2) = 2 \div 1,$$

and for $\Delta t_A = 6, (a = 6)$ and $\Delta t_B = 4, (b = 4)$ we may write

$$a \div b = 6 \div 4 = (3 \cdot 2) \div (2 \cdot 2) = 3 \div 2.$$

Table 2.1: List of generator combinations

Major	**Interference group (combination of generators)**						
3:	$3 \div 2$						
4:	$(2 \div 1)$	$4 \div 3$					
5:	$5 \div 2$	$5 \div 3$	$5 \div 4$				
6:	$(3 \div 1)$	$(2 \div 1)$	$(3 \div 2)$	$6 \div 5$			
7:	$7 \div 2$	$7 \div 3$	$7 \div 4$	$7 \div 5$	$7 \div 6$		
8:	$(4 \div 1)$	$8 \div 3$	$(2 \div 1)$	$8 \div 5$	$(4 \div 3)$	$8 \div 7$	
9:	$9 \div 2$	$(3 \div 1)$	$9 \div 4$	$9 \div 5$	$(3 \div 2)$	$9 \div 7$	$9 \div 8$

Now we will consider the combinations listed in the table and determine the resultant r. These will be shown in musical notation in the next chapter, after the aspect of grouping has been discussed; see Chapter 3.

Interference generators: 3 and 2

For $\Delta t_A = 3$ and $\Delta t_B = 2$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 3], \quad \vec{t_B} = [0\ 2\ 4],$$

repeating itself after $T_r = 3 \cdot 2 = 6$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 2\ 3\ 4], \quad r = 2 + 1 + 1 + 2.$$

Interference generators: 4 and 3

For $\Delta t_A = 4$ and $\Delta t_B = 3$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 4\ 8], \quad \vec{t_B} = [0\ 3\ 6\ 9],$$

repeating itself after $T_r = 4 \cdot 3 = 12$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 3\ 4\ 6\ 8\ 9], \quad r = 3 + 1 + 2 + 2 + 1 + 3.$$

Interference generators: 5 and 2

For $\Delta t_A = 5$ and $\Delta t_B = 2$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 5], \quad \vec{t_B} = [0\ 2\ 4\ 6\ 8],$$

repeating itself after $T_r = 5 \cdot 2 = 10$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 2\ 4\ 5\ 6\ 8], \quad r = 2 + 2 + 1 + 1 + 2 + 2.$$

Interference generators: 5 and 3

For $\Delta t_A = 5$ and $\Delta t_B = 3$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 5\ 10], \quad \vec{t_B} = [0\ 3\ 6\ 9\ 12],$$

repeating itself after $T_r = 5 \cdot 3 = 15$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 3\ 5\ 6\ 9\ 10\ 12], \quad r = 3 + 2 + 1 + 3 + 1 + 2 + 3.$$

Interference generators: 5 and 4

For $\Delta t_A = 5$ and $\Delta t_B = 4$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 5\ 10\ 15], \quad \vec{t_B} = [0\ 4\ 8\ 12\ 16],$$

repeating itself after $T_r = 5 \cdot 4 = 20$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 4\ 5\ 8\ 10\ 12\ 15\ 16], \quad r = 4 + 1 + 3 + 2 + 2 + 3 + 1 + 4.$$

Interference generators: 6 and 5

For $\Delta t_A = 6$ and $\Delta t_B = 5$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 6\ 12\ 18\ 24], \quad \vec{t_B} = [0\ 5\ 10\ 15\ 20\ 25],$$

repeating itself after $T_r = 6 \cdot 5 = 30$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 5\ 6\ 10\ 12\ 15\ 18\ 20\ 24\ 25], \quad r = 5 + 1 + 4 + 2 + 3 + 3 + 2 + 4 + 1 + 5.$$

Interference generators: 7 and 2

For $\Delta t_A = 7$ and $\Delta t_B = 2$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 7], \quad \vec{t_B} = [0\ 2\ 4\ 6\ 8\ 10\ 12],$$

repeating itself after $T_r = 7 \cdot 2 = 14$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 2\ 4\ 6\ 7\ 8\ 10\ 12], \quad r = 2 + 2 + 2 + 1 + 1 + 2 + 2 + 2.$$

Interference generators: 7 and 2

For $\Delta t_A = 7$ and $\Delta t_B = 3$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 7\ 14], \quad \vec{t}_B = [0\ 3\ 6\ 9\ 12\ 15\ 18],$$

repeating itself after $T_r = 7 \cdot 3 = 21$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t}_r = [0\ 3\ 6\ 7\ 9\ 12\ 14\ 15\ 18], \quad r = 3 + 3 + 1 + 2 + 3 + 2 + 1 + 3 + 3.$$

Interference generators: 7 and 4

For $\Delta t_A = 7$ and $\Delta t_B = 4$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 7\ 14\ 21], \quad \vec{t}_B = [0\ 4\ 8\ 12\ 16\ 20\ 24],$$

repeating itself after $T_r = 7 \cdot 4 = 28$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t}_r = [0\ 4\ 7\ 8\ 12\ 14\ 16\ 20\ 21\ 24], \quad r = 4 + 3 + 1 + 4 + 2 + 2 + 4 + 1 + 3 + 4.$$

Interference generators: 7 and 5

For $\Delta t_A = 7$ and $\Delta t_B = 5$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 7\ 14\ 21\ 28], \quad \vec{t}_B = [0\ 5\ 10\ 15\ 20\ 25\ 30],$$

repeating itself after $T_r = 7 \cdot 5 = 35$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t}_r = [0\ 5\ 7\ 10\ 14\ 15\ 20\ 21\ 25\ 28\ 30], \quad r = 5 + 2 + 3 + 4 + 1 + 5 + 1 + 4 + 3 + 2 + 5.$$

Interference generators: 7 and 6

For $\Delta t_A = 7$ and $\Delta t_B = 6$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 7\ 14\ 21\ 28\ 35], \quad \vec{t}_B = [0\ 6\ 12\ 18\ 24\ 30\ 36],$$

repeating itself after $T_r = 7 \cdot 6 = 42$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t}_r = [0\ 6\ 7\ 12\ 14\ 18\ 21\ 24\ 28\ 30\ 35\ 36], \quad r = 6 + 1 + 5 + 2 + 4 + 3 + 3 + 4 + 2 + 5 + 1 + 6.$$

Interference generators: 8 and 3

For $\Delta t_A = 8$ and $\Delta t_B = 3$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 8\ 16], \quad \vec{t}_B = [0\ 3\ 6\ 9\ 12\ 15\ 18\ 21],$$

repeating itself after $T_r = 8 \cdot 3 = 24$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t}_r = [0\ 3\ 6\ 8\ 9\ 12\ 15\ 16\ 18\ 21], \quad r = 3 + 3 + 2 + 1 + 3 + 3 + 1 + 2 + 3 + 3.$$

Interference generators: 8 and 5

For $\Delta t_A = 8$ and $\Delta t_B = 5$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 8\ 16\ 24\ 32], \quad \vec{t_B} = [0\ 5\ 10\ 15\ 20\ 25\ 30\ 35],$$

repeating itself after $T_r = 8 \cdot 5 = 40$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 5\ 8\ 10\ 15\ 16\ 20\ 24\ 25\ 30\ 32\ 35], \quad r = 5+3+2+5+1+4+4+1+5+2+3+5.$$

Interference generators: 8 and 7

For $\Delta t_A = 8$ and $\Delta t_B = 7$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 8\ 16\ 24\ 32\ 40\ 48], \quad \vec{t_B} = [0\ 7\ 14\ 21\ 28\ 35\ 42\ 49],$$

repeating itself after $T_r = 8 \cdot 7 = 56$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\vec{t_r} = [0\ 7\ 814\ 16\ 21\ 24\ 28\ 32\ 35\ 40\ 42\ 48\ 49], \quad r = 7{+}1{+}6{+}2{+}5{+}3{+}4{+}4{+}3{+}5{+}2{+}6{+}1{+}7.$$

Interference generators: 9 and 2

For $\Delta t_A = 9$ and $\Delta t_B = 2$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 9], \quad \vec{t_B} = [0\ 2\ 4\ 6\ 8\ 10\ 12\ 14\ 16],$$

repeating itself after $T_r = 9 \cdot 2 = 18$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\begin{aligned} \vec{t_r} &= [0\ 2\ 4\ 6\ 8\ 9\ 10\ 12\ 14\ 16], \\ r &= 2+2+2+2+1+1+2+2+2+2. \end{aligned}$$

Interference generators: 9 and 4

For $\Delta t_A = 9$ and $\Delta t_B = 4$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 9\ 18\ 27], \quad \vec{t_B} = [0\ 4\ 8\ 12\ 16\ 20\ 24\ 28\ 32],$$

repeating itself after $T_r = 9 \cdot 4 = 36$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\begin{aligned} \vec{t_r} &= [0\ 4\ 8\ 9\ 12\ 16\ 18\ 20\ 24\ 27\ 28\ 32], \\ r &= 4+4+1+3+4+2+2+4+3+1+4+4. \end{aligned}$$

Interference generators: 9 and 5

For $\Delta t_A = 9$ and $\Delta t_B = 5$ and using Eq. 1.2 we get the attack series

$$\vec{t_A} = [0\ 9\ 18\ 27\ 36], \quad \vec{t_B} = [0\ 5\ 10\ 15\ 20\ 25\ 30\ 35\ 40],$$

repeating itself after $T_r = 9 \cdot 5 = 45$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\begin{aligned} \vec{t_r} &= [0\ 5\ 9\ 10\ 15\ 18\ 20\ 25\ 27\ 30\ 35\ 36\ 40], \\ r &= 5+4+1+5+3+2+5+2+3+5+1+4+5. \end{aligned}$$

Interference generators: 9 and 7

For $\Delta t_A = 9$ and $\Delta t_B = 7$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 9\ 18\ 27\ 36\ 45\ 54], \quad \vec{t}_B = [0\ 7\ 14\ 21\ 28\ 35\ 42\ 49\ 56],$$

repeating itself after $T_r = 9 \cdot 7 = 63$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\begin{aligned} \vec{t}_r &= [0\ 7\ 9\ 14\ 18\ 21\ 27\ 28\ 35\ 36\ 42\ 45\ 49\ 54\ 56] \\ r &= 7+2+5+4+3+6+1+7+1+6+3+4+5+2+7. \end{aligned}$$

Interference generators: 9 and 8

For $\Delta t_A = 9$ and $\Delta t_B = 8$ and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 9\ 18\ 27\ 36\ 45\ 54\ 63], \quad \vec{t}_B = [0\ 8\ 16\ 24\ 32\ 40\ 48\ 56\ 64],$$

repeating itself after $T_r = 9 \cdot 8 = 72$ time units. We determine the resultant from the combination of these two attack series, Eq. 2.1, which yields the following attack and duration series

$$\begin{aligned} \vec{t}_r &= [0\ 8\ 9\ 16\ 18\ 24\ 27\ 32\ 36\ 40\ 45\ 48\ 54\ 56\ 63\ 64] \\ r &= 8+1+7+2+6+3+5+4+4+5+3+6+2+7+1+8. \end{aligned}$$

2.2 Grouping

Grouping is the selection of the *meter* or *time signature*; the resultant will be divided into a number of *measures* with each measure containing T time units.

In the case of binary synchronisation there are three options for grouping, discussed in the following subsections..

> Comment:
> Note that the next chapter, Ch. 3, is completely devoted to the aspect of grouping. Also note that apart from the three options mentioned here, Schillinger also considers what he calls *alien measure grouping* (see [3], Ch. **??**, p. 33).

2.2.1 Grouping by the common product

The *common product* is the number of time units after which the resultant attack pattern repeats itself. So, grouping by the common product implies

$$T = \Delta t_A \cdot \Delta t_B = ab\,\Delta t. \tag{2.3}$$

> Comment:
> Note that we will get unfamiliar time signatures, when we reach the higher numbers for the generators. Therefore, on [3], p. 10, there is an upper limit to this grouping technique: do not group the resultant by the common product when $T > 15$.

2.2.2 Superimposition of a

Superimposition of a means that the major generator A will determine the time signature

$$T = \Delta t_A = a\,\Delta t. \tag{2.4}$$

2.2.3 Superimposition of b

Superimposition of b means that the minor generator B will determine the time signature

$$T = \Delta t_B = b\,\Delta t. \tag{2.5}$$

2.3 Characteristics of the resultant

Although in the book this is not a separate section, Schillinger concludes the section with a few remarks on the characteristics of the resultant, summarised here as:

- The *recurrence* of the full pattern (repeating itself) after $T = \Delta t_A \cdot \Delta t_B = ab\,\Delta t$ time units.
- The rhythmic *symmetry* of the duration series about the centre (beginning and end are mirrored about the middle of the series).
- The use of both complete and incomplete resultants in music.
- The analogy with *cross-rhythm* when two generators create an interference pattern.

On [3], p. 9, there is a comment about the combined use of all the groupings, which Schillinger calls the *natural nucleus* of a musical score. There he writes that:

1. the duration pattern with the smallest time units Δt (the *common denominator*) may be used for *arpeggio* or *obligato* figures;
2. the resultants, grouped by either the major generator a or the minor generator b may be used for *chords* (harmonic structures);
3. the resultant $r_{a \div b}$ represents the rhythmic structure of the *melody*;
4. and the longest duration note with length $ab\,\Delta t$ may be used for sustained notes, such as a *pedal point*.

△ **Example 2.1.** The natural nucleus of a musical score

▷ This process is illustrated in Fig. 2.1, where a single chord is used to generate all components (melody, chords, pedal and arpeggio) from a score fragment. Note the small liberty in the duration of the attacks, and the slight rhythmic modification of the arpeggio on the right (the techniques from these books are not meant to be used rigidly, but to provide tools and stimulate creativity).

Figure 2.1: The natural nucleus of a musical score combining all the groupings. These represent the melody (M, grouped by the resultant r) the harmonic structures (HS, grouped by both a and b), the arpeggios (Arp, based on the smallest time unit) and the pedal point (Ped, grouped by the common product ab). On the left we see $4 \div 3$ grouped by $a = 4$, on the right we see $5 \div 3$ grouped by $b = 3$ (quarter note is 1 time unit).

Chapter 3

The techniques of grouping

Keywords: basic technique, grouping, meter, time signature.

This chapter focusses on the *grouping* of the resultant $r_{a \div b}$. A table is presented with practical *time signatures* for the binary generator synchronisation combinations listed in Table 2.1.

We briefly repeat the grouping options from Section 2.2 of the interference resultant $r_{a \div b}$ (a is the major, b the minor generator for binary synchronisation):

1. Grouping by the common product ab. The time signature for 1 measure T contains $T = ab$ time units. Use this time signature only for reasonable values of ab (practical limit: $ab < 15$).

2. Grouping by the major generator a (previously called superimposition by a). The fragment contains bT measures, and we will get the rhythmic effect of *syncopation*.

3. Grouping by the minor generator b. Now we get aT measures.

The grouping of the 19 combinations from Table 2.1 is shown in musical notation in Fig. 3.1 and 3.2. We use the time signatures from [3], p. 14 (Fig. 24).

Verify that grouping by the common product is only shown for values $ab < 15$; time signatures for larger values will hamper reading of the musical notation. Note the rhythmic symmetry about the middle of the series of durations. The differences in durations (short-long) are maximum at either beginning or end of the series, with more even durations in the middle. For grouping by either the major or the minor generator, check the number of measures and note that there are no slurred notes across bar lines.

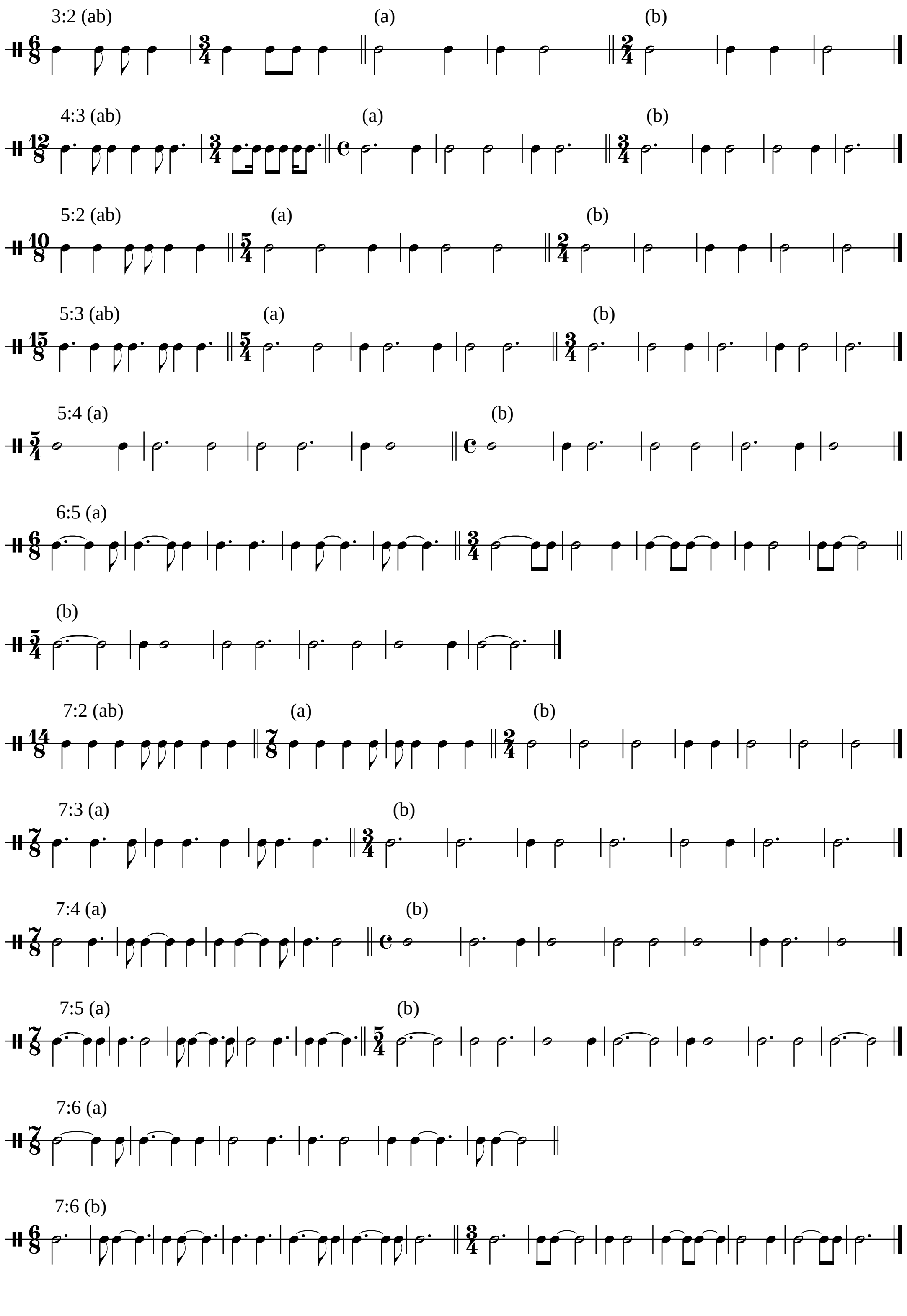


Figure 3.1: Grouping of non-uniform interference patterns

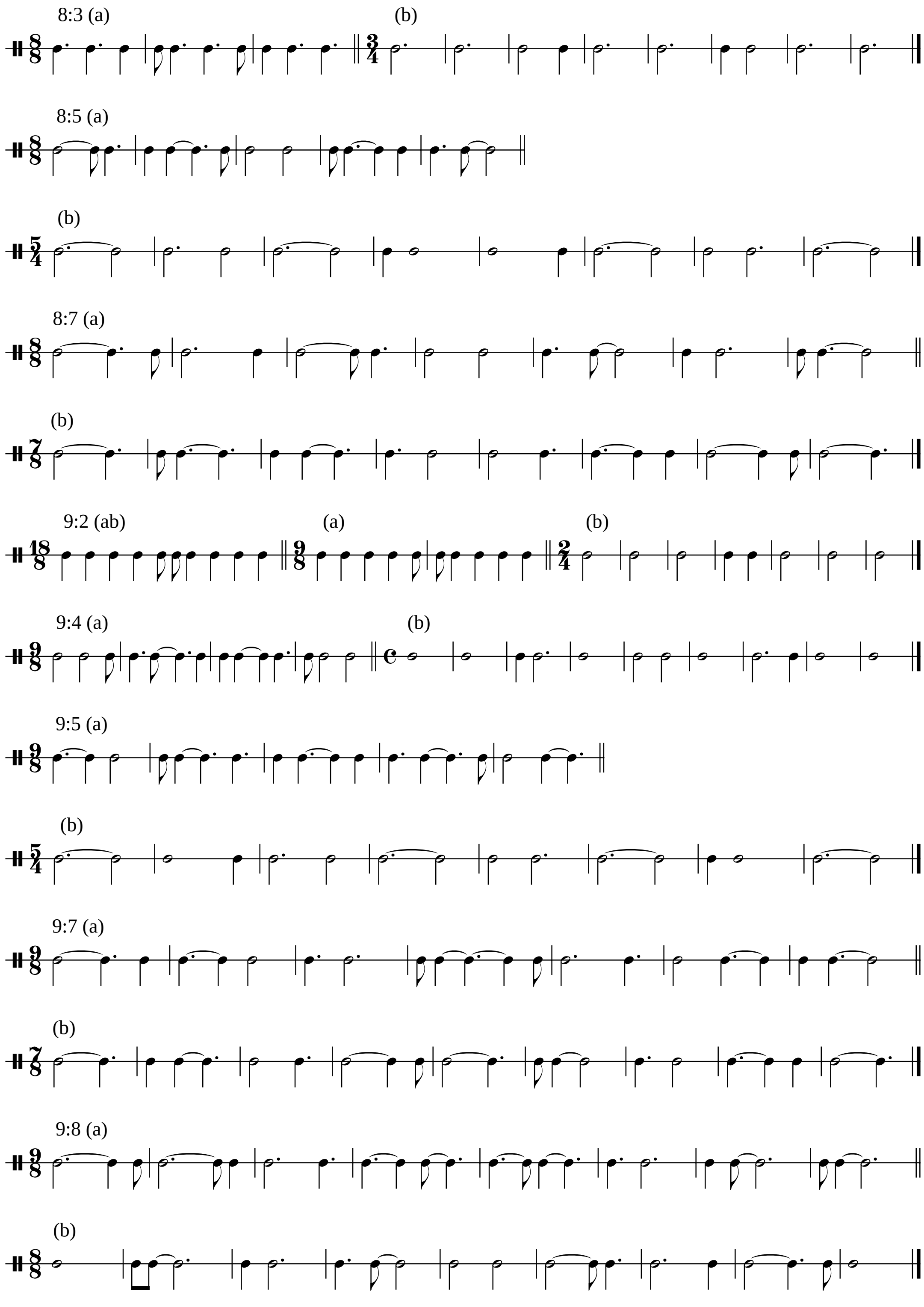


Figure 3.2: Grouping of non-uniform interference patterns (continued)

Chapter 4

The techniques of fractioning

Keywords: basic technique, fractioning, interference, notation, number theory.

This chapter introduces another basic technique for generating attack series with two generators a and b, called fractioning and notated as $\underline{a \div b}$. The interference and the grouping process are discussed.

4.1 The process of fractioning

The process of *fractioning* is again based on two generators, the major generator a and and the minor generator b (two integer numbers, with $a > b$). Again, we will use the analogy of ticking clocks or metronomes; in this case we will use 1 metronome of type A, ticking at intervals of Δt_A time units, and using N_b metronomes of type B, each generating a ticks at a time interval of Δt_B time units, with

$$N_b = a - b + 1. \tag{4.1}$$

We will synchronise these metronomes by starting the first B metronome at the same instant as metronome A. At each subsequent tick of metronome A we start another metronome B, until all N_b metronomes are ticking. The resultant attack series $\vec{t_r}$ is determined by the combination, the union, of all metronomes (compare this with Eq. 2.1 for binary synchronisation)

$$\vec{t_r} = \vec{t_A} \cup \vec{t_{B_1}} \cup \vec{t_{B_2}} \cup \ldots \cup \vec{t_{B_{N_b}}}, \tag{4.2}$$

and the duration series $r_{\underline{a \div b}}$ will repeat itself after $T = (\Delta t_A)^2$ time units.

The combinations for fractioning are listed in Table 2.1; here we will consider a number of examples and determine the resultant $r_{\underline{a \div b}}$. Not all combinations from the table will be considered here. See the example in Section 4.2 for the musical notation.

4.1.1 Fractioning group: 3 and 2

For $a = 3$ and $b = 2$, the interference pattern $\underline{3 \div 2}$ will repeat after $a^2 = 9$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 3 - 2 + 1 = 2$. Each B clock will generate $a = 3$ ticks. The attack series for the A and B generators are

$$\begin{aligned} \vec{t_A} &= [0\ 3\ 6], \\ \vec{t_{B_1}} &= [0\ 2\ 4], \\ \vec{t_{B_2}} &= [3\ 5\ 7]. \end{aligned}$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 2\ 3\ 4\ 5\ 6\ 7],$$
$$r = 2+1+1+1+1+1+2.$$

4.1.2 Fractioning group: 4 and 3

For $a = 4$ and $b = 3$, the interference pattern $\underline{4 \div 3}$ will repeat after $a^2 = 16$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 4 - 3 + 1 = 2$. Each B clock will generate $a = 4$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 4\ 8\ 12],$$
$$\vec{t}_{B_1} = [0\ 3\ 6\ 9],$$
$$\vec{t}_{B_2} = [4\ 7\ 10\ 13].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 3\ 4\ 6\ 7\ 8\ 9\ 10\ 12\ 13],$$
$$r = 3+1+2+1+1+1+1+2+1+3.$$

4.1.3 Fractioning group: 5 and 2

For $a = 5$ and $b = 2$, the interference pattern $\underline{5 \div 2}$ will repeat after $a^2 = 25$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 5 - 2 + 1 = 4$. Each B clock will generate $a = 5$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 5\ 10\ 15\ 20],$$
$$\vec{t}_{B_1} = [0\ 2\ 4\ 6\ 8],$$
$$\vec{t}_{B_2} = [5\ 7\ 9\ 11\ 13],$$
$$\vec{t}_{B_3} = [10\ 12\ 14\ 16\ 18],$$
$$\vec{t}_{B_4} = [15\ 17\ 19\ 21\ 23].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 2\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 23],$$
$$r = 2+2+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+2+2.$$

4.1.4 Fractioning group: 5 and 3

For $a = 5$ and $b = 3$, the interference pattern $\underline{5 \div 3}$ will repeat after $a^2 = 25$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 5 - 3 + 1 = 3$. Each B clock will generate $a = 5$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 5\ 10\ 15\ 20],$$
$$\vec{t}_{B_1} = [0\ 3\ 6\ 9\ 12],$$
$$\vec{t}_{B_2} = [5\ 8\ 11\ 14\ 17],$$
$$\vec{t}_{B_3} = [10\ 13\ 16\ 19\ 22].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 3\ 5\ 6\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 19\ 20\ 22],$$
$$r = 3+2+1+2+1+1+1+1+1+1+1+1+1+2+1+2+3.$$

4.1.5 Fractioning group: 5 and 4

For $a = 5$ and $b = 4$, the interference pattern $\underline{5 \div 4}$ will repeat after $a^2 = 25$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 5 - 4 + 1 = 2$. Each B clock will generate $a = 5$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 5\ 10\ 15\ 20],$$
$$\vec{t}_{B_1} = [0\ 4\ 8\ 12\ 16],$$
$$\vec{t}_{B_2} = [5\ 9\ 13\ 17\ 21].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 4\ 5\ 8\ 9\ 10\ 12\ 13\ 15\ 16\ 17\ 20\ 21],$$
$$r = 4+1+3+1+1+2+1+2+1+1+3+1+4.$$

4.1.6 Fractioning group: 6 and 5

For $a = 6$ and $b = 5$, the interference pattern $\underline{6 \div 5}$ will repeat after $a^2 = 36$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 6 - 5 + 1 = 2$. Each B clock will generate $a = 6$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 6\ 12\ 18\ 24\ 30],$$
$$\vec{t}_{B_1} = [0\ 5\ 10\ 15\ 20\ 25],$$
$$\vec{t}_{B_2} = [6\ 11\ 16\ 21\ 26\ 31].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\vec{t}_r = [0\ 5\ 6\ 10\ 11\ 12\ 15\ 16\ 18\ 20\ 21\ 24\ 25\ 26\ 30\ 31],$$
$$r = 5+1+4+1+1+3+1+2+2+1+3+1+1+4+1+5.$$

4.1.7 Fractioning group: 7 and 3

For $a = 7$ and $b = 3$, the interference pattern $\underline{7 \div 3}$ will repeat after $a^2 = 49$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 7 - 3 + 1 = 5$. Each B clock will generate $a = 7$ ticks. The attack series for the A and B generators are

$$\vec{t}_A = [0\ 7\ 14\ 21\ 28\ 35\ 42],$$
$$\vec{t}_{B_1} = [0\ 3\ 6\ 9\ 12\ 15\ 18],$$
$$\vec{t}_{B_2} = [7\ 10\ 13\ 16\ 19\ 22\ 25],$$
$$\vec{t}_{B_3} = [14\ 17\ 20\ 23\ 26\ 29\ 32],$$

$$\vec{t}_{B_4} = [21\ 24\ 27\ 30\ 33\ 36\ 39],$$
$$\vec{t}_{B_5} = [28\ 31\ 34\ 37\ 40\ 43\ 46].$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\begin{aligned}\vec{t}_r &= [0\ 3\ 6\ 7\ 9\ 10\ 13\ 14\ 15\ 17\ 19\ 20\ 21\ 22\ 23\ 24\ 25\\ &\quad 26\ 27\ 28\ 29\ 30\ 31\ 33\ 34\ 35\ 36\ 37\ 39\ 40\ 42\ 43\ 46],\\ r &= 3+3+1+2+1+3+1+1+2+2+1+1+1+1+1+1+\\ &\quad 1+1+1+1+1+1+1+2+2+1+1+3+1+2+1+3+3.\end{aligned}$$

4.1.8 Fractioning group: 7 and 4

For $a = 7$ and $b = 4$, the interference pattern $\underline{7 \div 4}$ will repeat after $a^2 = 49$ time units and we determine the number of B minor generators using Eq. 4.1; this yields $N_b = 7 - 4 + 1 = 4$. Each B clock will generate $a = 7$ ticks. The attack series for the A and B generators are

$$\begin{aligned}\vec{t}_A &= [0\ 7\ 14\ 21\ 28\ 35\ 42],\\ \vec{t}_{B_1} &= [0\ 4\ 8\ 12\ 16\ 20\ 24],\\ \vec{t}_{B_2} &= [7\ 11\ 15\ 19\ 23\ 27\ 31],\\ \vec{t}_{B_3} &= [14\ 18\ 22\ 26\ 30\ 34\ 38],\\ \vec{t}_{B_4} &= [21\ 25\ 29\ 33\ 37\ 41\ 45].\end{aligned}$$

Applying Eq. 4.2 the resultant attack and duration series are

$$\begin{aligned}\vec{t}_r &= [0\ 4\ 7\ 8\ 11\ 12\ 14\ 15\ 16\ 18\ 19\ 20\ 21\ 22\ 23\ 24\\ &\quad 25\ 26\ 27\ 28\ 29\ 30\ 31\ 33\ 34\ 35\ 37\ 38\ 41\ 42\ 45],\\ r &= 4+3+1+3+1+2+1+1+2+1+1+1+1+1+1+1+\\ &\quad 1+1+1+1+1+1+1+2+1+1+2+1+3+1+3+4.\end{aligned}$$

4.2 Grouping

Grouping is the selection of the *meter* or *time signature*; the resultant will be divided into a number of *measures* with each measure containing T time units.

In the case of fractioning there are again three options for grouping. The resultant attack pattern repeats itself after $(\Delta t_A)^2$ time units. So, grouping by a^2 implies

$$T = (\Delta t_A)^2 = a^2\,\Delta t. \tag{4.3}$$

Comment:
Like the case for binary synchronisation (see Section 2.2), there is an upper limit to this grouping technique: do not group the resultant by a^2 when $T > 15$.

Superimposition of a means that the major generator A will determine the time signature

$$T = \Delta t_A = a\,\Delta t. \tag{4.4}$$

Superimposition of b means that the minor generator B will determine the time signature

$$T = \Delta t_B = b\,\Delta t. \tag{4.5}$$

This will yield syncopated rhythms for the fractioning attack series, when we repeat the grouped resultant attack series until we stop at a complete measure, i.e., the attack series is completed at the end of a full measure, just before the bar line.

△ **Example 4.1.** Grouping of fractioning patterns

▷ A number of groupings of the resultant fractioning patterns discussed in Section 4.1 are shown in musical notation in Fig. 4.1. The smallest time unit is either the 8-th or the 4-th note duration. Note that the grouping by either a^2 or a again leads to symmetric attack patterns. Grouping the resultant by b leads to syncopated patterns; the end of each resultant pattern is indicated by the breathing sign ($'$).

Figure 4.1: Grouping of fractioning patterns

Chapter 5

Composition of groups by pairs

Keywords: rhythmic resultants, group pairing, balancing, expanding, contracting.

In this chapter the process of composing rhythmic resultants in pairs. Adjacent groups, based on the interference of two generators a and b. Uses a combination of the techniques described in Section 2.1 (binary synchronisation) and Section 4.1 (fractioning).

Binary synchronisation yields the resultant $r_{a\div b}$, fractioning generates the resultant $r_{\underline{a\div b}}$.

> Comment:
> Based on the basic techniques, presented in previous chapters, the pairing of resultants is a method to generate a longer rhythmic continuity. The unifying element is the set of two generators a and b, that will determine the note durations.

There are three approaches to the combination of these resultants, labeled as */balance*, *expansion*, and *contraction*. For all three approaches grouping by a time units only.

5.1 Balancing adjacent groups

A balanced pairing is given by

$$r_B(a,b) = +r_{\underline{a\div b}} + r_{a\div b} + a(a-b). \tag{5.1}$$

The balancing resultant sound unnatural when $a > mb$, with $m \geq 2$. In that case a balanced pairing is achieved by using

$$r_B(a > mb) = r_{\underline{a\div b}} + m\, r_{a\div b} + (a^2 - mab)\,\Delta t. \tag{5.2}$$

△ **Example 5.1.** Balancing resultant: $r_B(a,b)$

▷ For the following examples we will present the duration series, using Eq. 5.1 or Eq. 5.2, and the results from Section 2.1.2 (interference through non-uniform binary synchronisation) and Section 4.1 (fractioning). The resultants are shown in musical notation in Fig. 5.1.

▷ Generator time units $a = 3, b = 2$.

$$\begin{aligned} r_B(3,2) &= r_{3\div 2} + r_{\underline{3\div 2}} + 3(3-2) \\ &= (2+1+1+2) + (2+1+1+1+1+1+2) + 3. \end{aligned}$$

▷ Generator time units $a = 3, b = 2$.

$$\begin{aligned} r_B(4,3) &= r_{4\div 3} + r_{\underline{4\div 3}} + 4(4-3) \\ &= (3+1+2+2+1+3)+ \\ &\quad +(3+1+2+1+1+1+1+2+1+3)+4. \end{aligned}$$

▷ Generator time units $a = 5, b = 3$.

$$\begin{aligned} r_B(5,3) &= r_{\underline{5\div 3}} + r_{5\div 3} + +5(5-3) \\ &= (3+2+1+2+1+1+1+1+1+ \\ &\quad +1+1+1+1+2+1+2+3)+ \\ &\quad +(3+2+1+3+1+2+3)+10. \end{aligned}$$

▷ Generator time units $a = 5, b = 2$.

$$\begin{aligned} r_B(5,2) &= r_{\underline{5\div 2}} + r_{5\div 2} + 5(5-2) \\ &= (2+2+1+1+1+1+1+1+1+1+1+ \\ &\quad +1+1+1+1+1+1+1+1+2+2)+ \\ &\quad +(3+2+1+3+1+2+3)+15. \end{aligned}$$

▷ Generator time units $a = 7, b = 3$.

$$\begin{aligned} r_B(7,3) &= r_{\underline{7\div 3}} + r_{7\div 3} + +7(7-3) \\ &= (3+3+1+2+1+3+1+1+2+2+1+ \\ &\quad +1+1+1+1+1+1+1+1+1+1+1+ \\ &\quad +1+2+2+1+1+3+1+2+1+3+3)+ \\ &\quad +(3+3+1+2+3+2+1+3+3)+28. \end{aligned}$$

5.2 Expanding adjacent groups

The expanding pairing is given by

$$r_E(a,b) = r_{a\div b} + r_{\underline{a\div b}}. \tag{5.3}$$

△ **Example 5.2.** Expanding resultant: $r_E(a,b)$

▷ For the following examples we will present the duration series, using Eq. 5.3, and the results from Section 2.1.2 and Section 4.1. The resultants are shown in musical notation in Fig. 5.1.

▷ Generator time units $a = 3, b = 2$.

$$\begin{aligned} r_E(3,2) &= r_{3\div 2} + r_{\underline{3\div 2}} \\ &= (2+1+1+2)+(2+1+1+1+1+1+2). \end{aligned}$$

▷ Generator time units $a = 5, b = 3$.

$$\begin{aligned} r_E(5,3) &= r_{5\div 3} + r_{\underline{5\div 3}} \\ &= (3+2+1+3+1+2+3)+ \\ &\quad +(3+2+1+2+1+1+1+1+1+ \\ &\quad +1+1+1+1+2+1+2+3). \end{aligned}$$

5.3 Contracting adjacent groups

A contracting pairing is given by

$$r_C(a,b) = r_{\underline{a\div b}} + r_{a\div b}. \tag{5.4}$$

△ **Example 5.3.** Contracting resultant: $r_C(a,b)$

▷ For the following examples we will present the duration series, using Eq. 5.4, and the results from Section 2.1.2 and Section 4.1. The resultants are shown in musical notation in Fig. 5.1.

▷ Generator time units $a = 4, b = 3$.

$$\begin{aligned} r_C(4,3) &= r_{\underline{4\div 3}} + r_{4\div 3} \\ &= (3+1+2+1+1+1+1+2)+(3+1+2+2+1+3). \end{aligned}$$

▷ Generator time units $a = 5, b = 3$.

$$\begin{aligned} r_C(5,3) &= r_{\underline{5\div 3}} + r_{5\div 3} \\ &= (3+2+1+2+1+1+1+1+1+ \\ &\quad +1+1+1+1+2+1+2+3)+ \\ &= (3+2+1+3+1+2+3). \end{aligned}$$

Note from the examples, that:

- The second group of the resultant pair always is a varied version of the first group. This created both unity and variation.
- In balancing (r_B) and contracting (r_C) the pair, there is more rhythmic activity, i.e., shorter note durations, in the first half of the resultant. The balanced pairing concludes with a very long duration note.
- When the difference between the two generators becomes large, such as in $r_B(7,3)$, indeed the resultant has unnatural characteristics, such as long series of equal short notes in the first half, and a very long closing note. This was remarked by Schillinger as the special case for $a > mb$ (see above).
- The expanding pair (r_E) has more rhythmic activity (shorter notes) in the second half of the resultant.

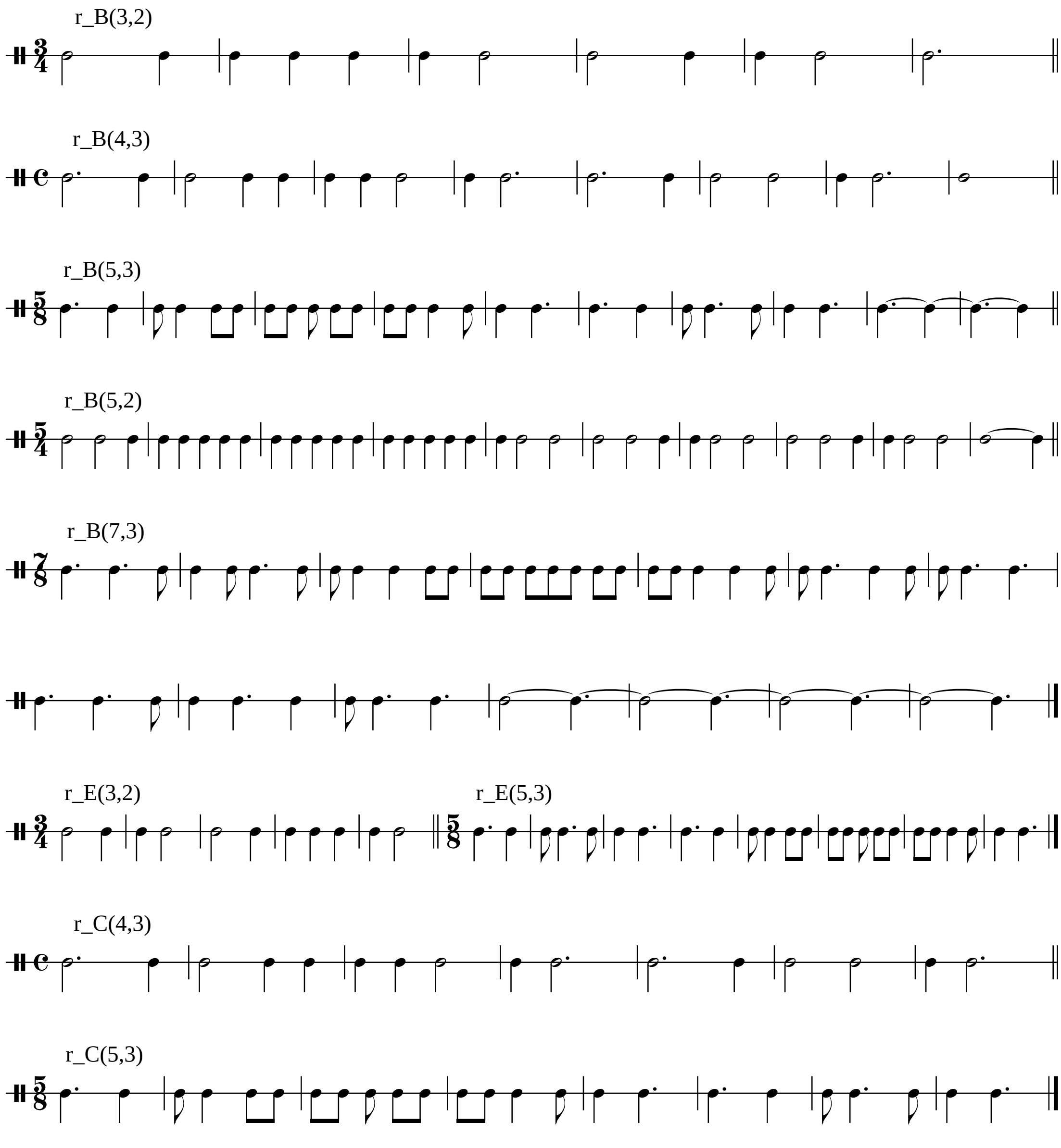


Figure 5.1: Balancing (r_B), expanding (r_E) and contracting (r_C) a pair of adjacent groups.

Chapter 6

Utilization of three or more generators

Keywords: basic technique, interference.

This chapter introduces a basic technique for generating attack series, by considering the interference pattern of more than two clocks or metronomes that each tick at a different interval. These attack series may then be grouped using different numbers of time units per measure.

The three generators will form a *family of rhythms* when they are based on the same series of growth, also called *summation series* or *Fibonacci series*, shown in Table 6.1.

Table 6.1: The summation series useful for musical purposes. Each row in the table is a Fibonacci summation series with each element being the sum of the two previous elements.

1	2	3	5	8	13	...
1	3	4	7	11	18	...
1	4	5	9	14	23	...

We will limit the combinations to practical size, and consider only the combinations discussed in the next section.

6.1 The technique of synchronisation of three generators

Now there are three clocks or metronomes A, B and C ticking at different regular time intervals Δt_A, Δt_B and Δt_C. We assume that metronome C is ticking faster than metronome B, and metronome B is ticking faster than metronome A. So we have $\Delta t_C < \Delta t_B < \Delta t_A$. 3 

Now we may determine two resultants r and r'. The interference pattern will repeat after $T_r = \Delta t_C \cdot \Delta t_B \cdot \Delta t_A$ time units. The resultant r is determined analogous to the case of binary synchronisation (see Section 2.1.2), by finding the combination (the union) of three attack series

$$\vec{t}_r = \vec{t}_A \cup \vec{t}_B \cup \vec{t}_C. \tag{6.1}$$

The alternative resultant r' is determined by synchronising the *complementary factors*: this is a set of three alternative generators, but now the metronomes tick at intervals of $\Delta t_A \cdot \Delta t_B$ (the complementary factor of generator C), $\Delta t_A \cdot \Delta t_C$ (the complementary factor of generator B), and $\Delta t_B \cdot \Delta t_C$ (the complementary factor of generator A).

Comment:
We will see in the worked-out examples that r yields shorter duration series than r', which creates rhythmic patterns with longer durations.

6.1.1 Interference group: 5, 3 and 2

For $\Delta t_A = 5,\ \Delta t_B = 3$ and $\Delta t_C = 2$ time units and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 5\ 10\ \ldots\ 25],\quad \vec{t}_B = [0\ 3\ 6\ \ldots\ 27],\quad \vec{t}_C = [0\ 2\ 4\ldots\ 28],$$

repeating itself after $T_r = 2 \cdot 3 \cdot 5 = 30$ time units. We determine the resultant r from the combination of these attack series, Eq. 6.1, which yields the following attack and duration series

$$\begin{aligned}\vec{t}_r &= [0\ 2\ 3\ 4\ 5\ 6\ 8\ 9\ 10\ 12\ 14\ 15\ 16\ 18\ 20\ 21\ 22\ 24\ 25\ 26\ 27\ 28],\\ r &= 2+1+1+1+1+2+1+1+2+2+1+\\ &\quad 1+2+2+1+1+2+1+1+1+1+2.\end{aligned}$$

The resultant r' is obtained with the complementary clocks, with $\Delta t_{A'} = 6,\ \Delta t_{B'} = 10$ and $\Delta t_{C'} = 15$ and using Eq. 1.2 we get the attack series

$$\vec{t}_{A'} = [0\ 6\ 12\ldots\ 24],\quad \vec{t}_{B'} = [0\ 10\ 20],\quad \vec{t}_{C'} = [0\ 15],$$

repeating itself after $T_r = 30$ time units. The combination of these attack series yields the following attack and duration series

$$\begin{aligned}\vec{t}_{r'} &= [0\ 6\ 10\ 12\ 15\ 18\ 20\ 24],\\ r' &= 6+4+2+3+3+2+4+6.\end{aligned}$$

6.1.2 Interference group: 7, 4 and 3

For $\Delta t_A = 7,\ \Delta t_B = 4$ and $\Delta t_C = 3$ time units and using Eq. 1.2 we get the attack series

$$\vec{t}_A = [0\ 7\ 14\ \ldots\ 77],\quad \vec{t}_B = [0\ 4\ 8\ \ldots\ 80],\quad \vec{t}_C = [0\ 3\ 6\ldots\ 81],$$

repeating itself after $T_r = 7 \cdot 4 \cdot 3 = 84$ time units. We determine the resultant r from the combination of these attack series, Eq. 6.1, which yields the following attack and duration series

$$\begin{aligned}\vec{t}_r &= [0\ 3\ 4\ 6\ 7\ 8\ 9\ 12\ 14\ 15\ 16\ 18\ 20\ 21\ 24\ 27\ 28\ 30\ 32\ 33\ 35\ 36\ 39\ 40\ 42\\ &\quad 44\ 45\ 48\ 49\ 51\ 52\ 54\ 56\ 57\ 60\ 63\ 64\ 66\ 68\ 69\ 70\ 72\ 75\ 76\ 77\ 78\ 80\ 81],\\ r &= 3+1+2+1+1+1+3+2+1+1+2+2+1+3+3+1+2+2+1+2+1+\\ &\quad 3+1+2+2+1+3+1+2+1+2+2+1+3+3+1+2+2+1+1+2+3+\\ &\quad 1+1+1+2+1+3.\end{aligned}$$

The resultant r' is obtained with the complementary clocks, with $\Delta t_{A'} = 12,\ \Delta t_{B'} = 21$ and $\Delta t_{C'} = 28$ and using Eq. 1.2 we get the attack series

$$\vec{t}_{A'} = [0\ 12\ 24\ldots\ 72],\quad \vec{t}_{B'} = [0\ 21\ 42\ 63],\quad \vec{t}_{C'} = [0\ 28\ 56],$$

repeating itself after $T_r = 84$ time units. The combination of these attack series yields the following attack and duration series

$$\begin{aligned}\vec{t}_{r'} &= [0\ 12\ 21\ 24\ 28\ 36\ 42\ 48\ 56\ 60\ 63\ 72],\\ r' &= 12+9+3+4+8+6+6+8+4+3+9+12.\end{aligned}$$

6.2 Grouping

Grouping now can be done by using either the generators or the complementary factors as measure units. So we get 6 alternative groupings with 1 measure containing $T = \Delta t_A,\ \Delta t_B,]; \Delta t_C = 2,\ \Delta t_B \times \Delta t_C,\ \Delta t_A \times \Delta t_C,\ \Delta t_A \times \Delta t_B$ time units. As before we will limit the grouping to a practical maximum limit of $T = 15$ time units.

For the interference group $2 \div 3 \div 5$ this yields grouping by either $T = 2,\ 3,\ 5,\ 6,\ 10,$ or 15 time units. The result is shown in musical notation in Fig. 6.1. For the interference group $3 \div 4 \div 7$ this yields grouping by either $T = 3,\ 4,\ 7,\ 12,$ or 12 time units, and the result is shown in Fig.6.2.

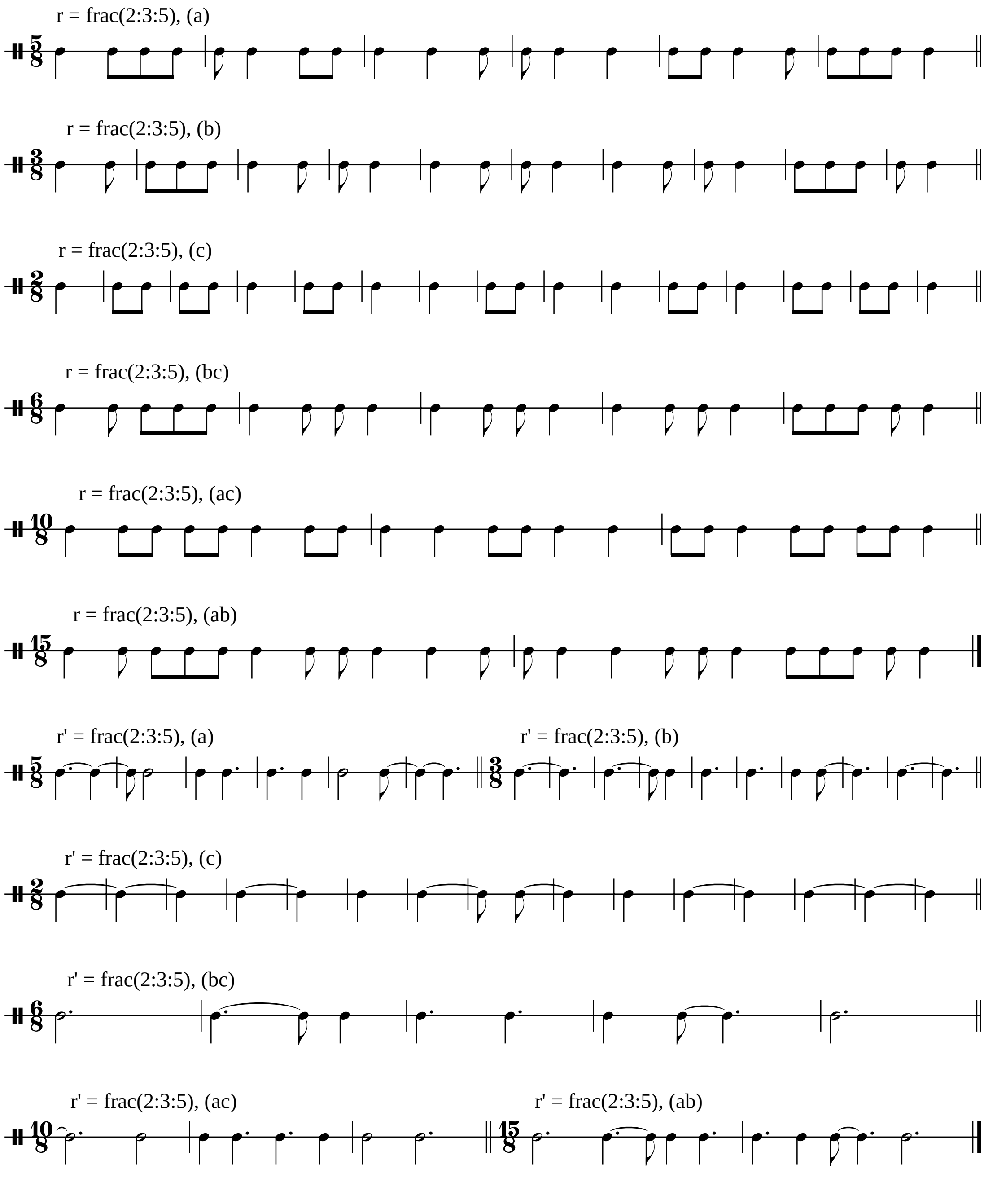


Figure 6.1: Grouping of fractioning patterns for interference group $2 \div 3 \div 5$

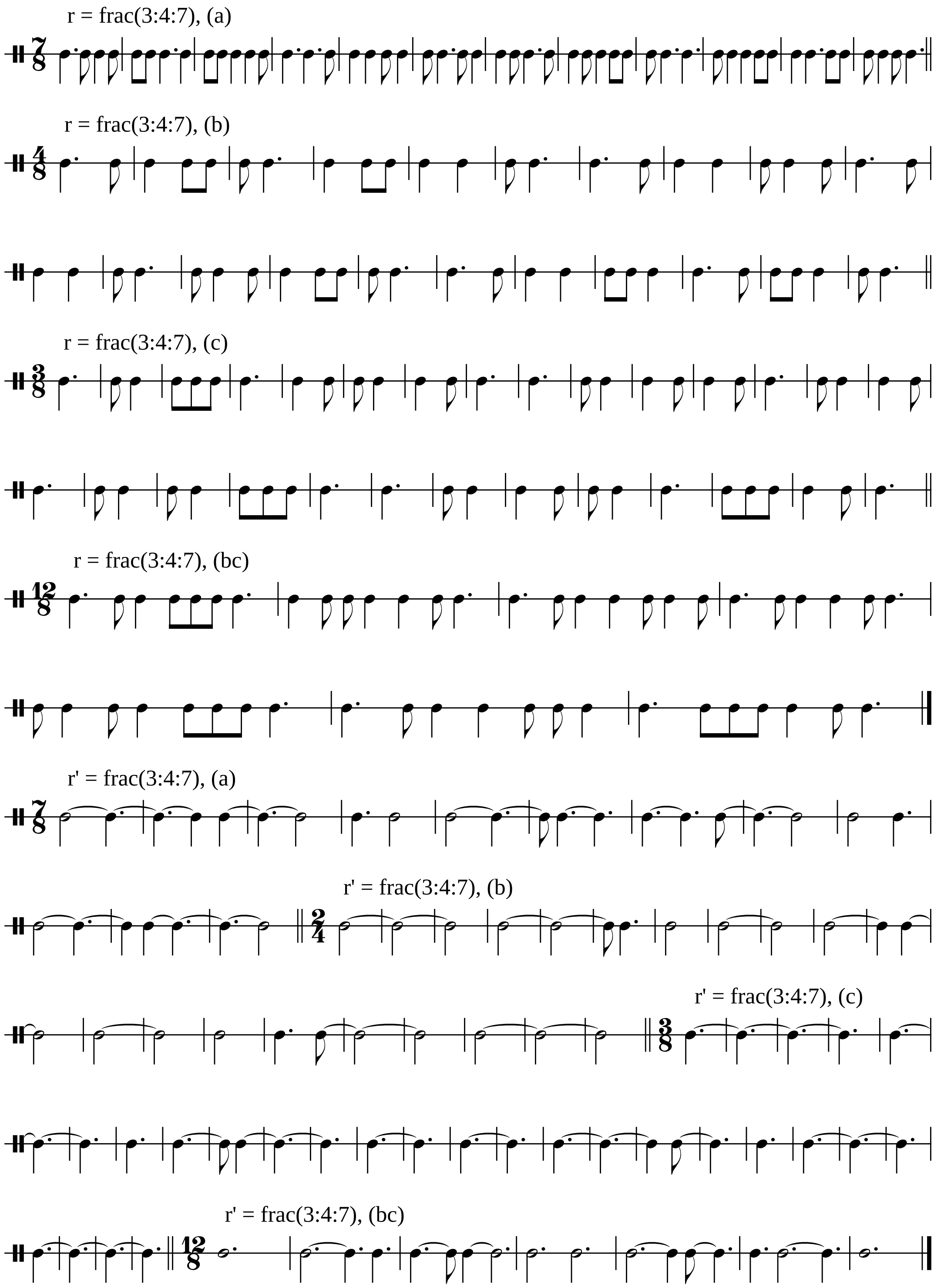


Figure 6.2: Grouping of fractioning patterns for interference group $3 \div 4 \div 7$

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