Twelve-tone chords

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Abstract

This document discusses musical composition with 12-tone chords. Harmonic structures are built from a set of basic 4-part chord structures (mainly chords in thirds), distributed over three parallel layers. Harmonic tension, i.e., the dissonance caused by minor 9ths, is minimized, adhering to a set of rules. This yields a limited number of possible 12-tone chords.

1 Introduction

Recently I began working on a composition assignment: a show that starts with picturing the evolution on planet Earth. Somehow I felt that traditional harmony could not do the trick, and I wanted something stronger, but still acceptable for a fairly traditional audience.

So I started looking at 12-tone chord structures, based on the idea that the twelve pitches from the chromatic scale represent the tone collection in the universe (not true, I know, but still). After the big bang a gradual development from the primordial boiling soup towards an ordered, habitable planet with land and sea had to be depicted. Chaos could be represented by 12-tone chord structures, from which order emerges as traditional harmony in thirds.

Use of 12-tone chord structures is discussed in a book about the music of Lutosławski, see [1]. I reread Chapter 3 from that book, and while carefully studying the matter, I discovered the beauty and logical process behind the derivation of the 12-tone chord structures, presented there. In fact, they are the only solution, when a number of basic rules for minimizing the harmonic tension is being respected. That logical process is illustrated in this document, with comments on the result and usefulness.

2 The starting point: 4-part chords in thirds

The 12-tone chord structures are constructed, using a set of traditional 4-part chord structures as building blocks, see Fig. 1 and Table 1. All chords share the property that the interval between adjacent pitches is either a perfect consonant (the perfect 4th = $5i$) or an imperfect consonant (major and minor thirds, i.e., $4i$ or $3i$).

Most of these are chords in thirds only, with a familiar flavour, originating from tonal harmony. The last three structures contain a perfect 4th and remind us of the augmented ninth $S^9_5$ or $S^9_7$ dominant structures, used frequently in jazz music. Working from this starting point a great number of other 12-tone vertical combinations are rejected, that would generate too many
Twelve-tone chords

Figure 1: Basic 4-part chord units: the first seven chords are chords in (major/minor) thirds, the last three contain a perfect 4th. These are the building blocks for generating 12-tone chords with minimum tension.

Table 1: The basic 4-part chord structures. The interval content is shown as the intervals between neighbouring pitches, expressed in numbers of semitones \( i \). The first seven chords are chords in (major/minor) thirds, the last three contain a perfect 4th.

<table>
<thead>
<tr>
<th>Label</th>
<th>Interval content</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{07} )</td>
<td>( 3i + 3i + 3i )</td>
<td>diminished 7th chord</td>
<td>( C_{07} )</td>
</tr>
<tr>
<td>( S_{07} )</td>
<td>( 3i + 3i + 4i )</td>
<td>half-diminished 7th chord</td>
<td>( C_{07} )</td>
</tr>
<tr>
<td>( S_{m7} )</td>
<td>( 3i + 4i + 3i )</td>
<td>minor 7th chord</td>
<td>( C_{m7} )</td>
</tr>
<tr>
<td>( S_{7} )</td>
<td>( 4i + 3i + 3i )</td>
<td>dominant 7th chord</td>
<td>( C_{7} )</td>
</tr>
<tr>
<td>( S_{m7}^{27} )</td>
<td>( 3i + 4i + 4i )</td>
<td>minor chord with major 7th</td>
<td>( C_{m7}^{27} )</td>
</tr>
<tr>
<td>( S_{7}^{27} )</td>
<td>( 4i + 3i + 4i )</td>
<td>major 7th chord</td>
<td>( C_{7}^{27} )</td>
</tr>
<tr>
<td>( S_{55}^{57} )</td>
<td>( 4i + 4i + 3i )</td>
<td>major 7th chord with augmented 5th</td>
<td>( C_{55}^{57} )</td>
</tr>
<tr>
<td>( S_{(5/3/3)} )</td>
<td>( 5i + 3i + 3i )</td>
<td>leading tone diminished triad above root</td>
<td>( F_{5}/C )</td>
</tr>
<tr>
<td>( S_{(3/5/3)} )</td>
<td>( 3i + 5i + 3i )</td>
<td>major triad with augmented 9th</td>
<td>( A_{b7}^{19} )</td>
</tr>
<tr>
<td>( S_{(3/3/5)} )</td>
<td>( 3i + 3i + 5i )</td>
<td>dominant 7th chord with augmented 9th</td>
<td>( A_{b7}^{19} )</td>
</tr>
</tbody>
</table>

dissonances between adjacent or nearby pitches, such as seconds = \( 2i \) (major second) or \( i \) (minor second), augmented 4th or diminished 5th, i.e., \( 6i \). These vertical orderings would yield an unacceptably high tension within in the 4-part chord structure.

3 Creating the 12-tone chord structures in three layers

We will generate the 12-tone chord structures by positioning three building blocks in three layers: Layer 1 is the bottom (bass), Layer 2 the middle and Layer 3 the top (high pitch) layer.

3.1 Three strata or layers with minimum tension

We will respect two rules for minimizing the tension in the 12-tone chord structure:

**Rule 1:** Use 4-part chords from the basic set presented in Table 1 or Fig. 1 in each layer.

**Rule 2:** Prevent or minimize the total number of intervals of the minor 9th between the pitches in adjacent layers.
Rule 1 assures that we are staying somewhat close to traditional harmony, as discussed above. Rule 2 obviously tells us to limit the number of what is regarded as the most harsh interval between two pitches, i.e., the minor 9th. These intervals should preferably occur between Layer 1 (bottom) and Layer 3 (top), in a wide spacing (separated by two or more octaves, if possible).

The total set of solutions for 12-tone chord structures is limited and can be achieved through applying the rules to the pitches in the chromatic scale. This total set for Group 1 and 2 is presented in [1], however, without mentioning the fact that they are the only solution, satisfying the rules for minimizing the tension. An overview of the possible 12-tone chord structures is shown in Table 2.

3.2 Group 1: identical chords in all layers

The 12-tone chord with identical 4-part chord structures in all layer has one solution only: it contains the diminished 7th chord $S_{\text{o}7}$ in all three layers, as shown in Fig. 2.a. The tone disc representation is shown in Fig. 3.a. The chord structure is symmetrical $S_{\text{o}7} = S_{(3/3/3)}$. The root $R_2$ in Layer 2 lies a semitone below the root $R_1$ in Layer 1, the root $R_3$ in Layer 3 a semitone above $R_1$; all intervals between corresponding pitches in Layer 1 and Layer 2 are major 7ths (11 semitones), all minor 9ths (13 semitones) occur between Layer 1 and Layer 3, and therefore are widely separated (note the $8\text{va}$ sign above the treble clef in Layer 1 in Fig. 2).

3.3 Group 2: identical chords in two layers

There are three solutions, when creating 12-tone chord structures with identical 4-part chords in 2 layers, as shown in Fig. 2.b. In the tone disc representation, these are shown in Fig. 3.b, 3.c and 3.d. The identical chords are placed in Layers 1 and 3, with the root $R_3$ in Layer 3 always
Figure 3: Tone disc representation of 12-tone chords. (a): Group 1, identical chords in all layers $S_{07}$ in Layer 1, 2, and 3. (b): Group 2, $S_{\emptyset 7}$ in Layer 1 and 3, $S_{(5/3/3)}$ in Layer 2. (c): Group 2, $S_{m7}$ in Layer 1 and 3, $S_{(3/5/3)}$ in Layer 2. (d): Group 2, $S_7$ in Layer 1 and 3, $S_{(3/3/5)}$ in Layer 2. Symbols: •: pitches in Layer 1, ◦: pitches in Layer 2, ⋆: pitches in Layer 3, $R_{1,2,3}$: layer roots.

![Tone disc representation of 12-tone chords](image)

Table 2: Categories of 12-tone chords

<table>
<thead>
<tr>
<th>Layer</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (top)</td>
<td>$S_{07}$</td>
<td>$S_{\emptyset 7}$</td>
<td>$S_{m7}$</td>
</tr>
<tr>
<td>2 (middle)</td>
<td>$S_{07}$</td>
<td>$S_{(5/3/3)}$</td>
<td>$S_{(3/5/3)}$</td>
</tr>
<tr>
<td>1 (bottom)</td>
<td>$S_{07}$</td>
<td>$S_{\emptyset 7}$</td>
<td>$S_{m7}$</td>
</tr>
</tbody>
</table>

1 semitone above the root in Layer 1; this guarantees the 4 dissonant intervals of the minor 9th to be widely separated between the outer layers. In the middle layer we find the $S_{29}$-like chord structure $S_{(5/3/3)}$, $S_{(3/5/3)}$ or $S_{(3/3/5)}$.

### 3.4 Group 3: different chord structure in each layer

There are two solutions, when creating 12-tone chord structures with different 4-part chords in each layer, as shown in Fig. 2.c. In the tone disc representation these are shown in Fig. 4.a and 4.b.

### 3.5 Different vertical ordering

The vertical ordering of the 4-part chords in the possible solutions, achieved in the previous sections, may be changed. This will affect the chord tension, however.

As an example, we will reconsider the situation for Group 2 (identical chord structure in 2 layers), as shown in Fig. 3.c, where $S_{m7}$ was used twice in the outer layers. Putting the other chord $S_{(3/5/3)}$ in either Layer 3 or Layer 1 yields the situation depicted in as shown in Fig. 2.d, Fig. 4.c and d. This vertical ordering creates more tension, since now there is the dissonant interval of the minor 9th between adjacent layers (note in Fig. 4.c the pitch number 10, $B_\#$, in Layer 1 with the pitch number 11, $B$ in Layer 2. The same thing happens in Fig. 4.d, with the pitch number 5, $F$ in Layer 2, and the pitch number 6, $F_{\#}$, in Layer 3). An overview of the
Figure 4: Tone disc representation of 12-tone chords. (a): Group 3, different chords in all layers, $S_{\Delta 7}$ in Layer 1, $S_{(3/5/3)}$ in Layer 2, and $S_{m7}$ in Layer 3. (b): Group 3, $S_{\Delta 7}^{35}$ in Layer 1, $S_{\Delta 7}$ in Layer 2, and $S_{m7}^{52}$ in Layer 3. (c): Group 2, different vertical ordering, $S_{m7}$ in Layer 1 and 2, $S_{(3/5/3)}$ in Layer 3. (d): Group 2, different vertical ordering, $S_{(3/5/3)}$ in Layer 1, $S_{m7}$ in Layer 2 and 3. Symbols: •: pitches in Layer 1, ◦: pitches in Layer 2, ⋆: pitches in Layer 3, $R_{1,2,3}$: Layer roots.

Table 3: Group 2, different vertical ordering (example for $S_{m7}$).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (top)</td>
<td>$S_{m7}$</td>
</tr>
<tr>
<td>2 (middle)</td>
<td>$S_{(3/5/3)}$</td>
</tr>
<tr>
<td>1 (bottom)</td>
<td>$S_{m7}$</td>
</tr>
</tbody>
</table>

vertical ordering options for Group 2 is presented in Table 3.

**Question:** From the set of building blocks (the basic 4-part chords) one chord in thirds never returned as the fundamental chord in the bottom layer. Which chord is that and why was it not considered any further?

**Answer:** It is the chord structure $S_{m7}^{35}$. There is no Group 2 solution (the structure occurring twice). A Group 3 (3 different chords) solution does exist but is equivalent to a different vertical ordering of the solution with $S_{\Delta 7}^{35}$ in the bottom layer. This ordering is $S_{m7}^{35}$ with root $R_1 = 0$ in Layer 1 (bottom), $S_{\Delta 7}^{35}$ with root $R_2 = 6$ in Layer 2 (middle) and $S_{\Delta 7}$ with root $R_3 = 9$ in Layer 3 (top), i.e., starting from the bottom: $C^7_{m} + F^6_{\Delta 7} + A_{\Delta 7}$. (Check this answer!)

## 4 Root progression

We will consider the following root progression (in Layer 1): $C - G - F - D♭ - C$, which represents a progression: tonic - dominant - subdominant - dominant - tonic on root $C$ and a deceptive cadence between the second and third chord.
The result is shown in Fig. 5. Note the close vs. open voicings in the layers and the motion between the outer voices. In the example the chord position in the bottom layer is varied; the first and third chord are in first inversion, stressing the interval of the 6th between the 3 and root of the chord for less stability, see Chapter 18 from [3]. The last two chords are in root position, with the root doubled at the lower octave. The final chord also stresses the perfect 5th in the bass (between root and 5) as a stabilising factor.

For voice leading aspects and options with chord structures in three strata or layers, see Book IX from the Schillinger System of Musical Composition [2].

5 Conclusion

And so I found the solution to my compositional challenge of picturing the evolution process of our planet Earth; after a number of fortissimo tutti accents (the big bang) and busy arpeggios (the boiling soup) based on a 12-tone chord progression, the music became more tranquil and the traditional chords in thirds in each layer emerged separately (as long stable chords), thereby realising the transition to a section with modal harmonies. Of course, careful orchestration is required in this process.

The logic behind the derivation of the 12-tone chord structures with minimum tension became evident while studying the subject and was worth while to put down on paper. It leads to a useful technique in the toolbox of the contemporary composer/arranger who likes to work on the edge between jazz and contemporary classical music styles.

Document history

References

