Pitch-Class Set Diagrams

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Abstract

This document contains blank diagrams that may be used for composing atonal music based on pitch-class sets. The origin and use of these diagrams is explained in the text.

1 Introduction

Structures (melodic or harmonic) in *atonal music* are based on *pitch-class sets*. These are sets of tones from the 12-pitch *chromatic scale*. In the chromatic scale each pair of consecutive tones forms the interval of a *semitone*. In *dodecaphonic music* series of 12 tones is constructed from the same collection, with each pitch from the full collection occurring only once.

Pitch-class sets can be categorised: there is a bounded collection of sets with $N = 3, 4, 5$ or 6 pitches (the complement of that collection is formed by sets with $N = 7, 8$ or 9 pitches). I use the classification presented in [2]. Composing music with pitch-class sets involves mathematical operations on these sets. This subject is extensively treated in a number of textbooks, such as [2, 4, 5]. The most obvious operations are *transposition* and *inversion* of the original set. Pitch-class sets are also useful in *musical analysis* of 20th-century music; see [1, 3] for an overview and a number of books on the works of specific composers, such as Strawinsky, Bartók, Schoenberg, Berg and Webern.

In order to compose music with pitch-class sets or understand the mathematical operations, the aspect of *representation* becomes relevant; in the literature we find several systems for representation, either with unique labels or in graphical form.

2 Representations of pitch-class sets

Pitch-class sets use integers $p = \{0, 1, \ldots, 11\}$ to represent the collection of pitches from the 12-tone chromatic scale.

We may use two systems for representation:

1. The *Prime Form*: each pitch set $[p_1, \ldots, p_N]$ is labeled with two numbers separated by a hyphen, e.g., 4-14. The first number indicates the total number of pitches in the set ($N = 4$ in the example), the second number gives the position in an ordered list, as described in [2]. The pitch-class set 4-14 in the Prime Form contains the following pitches: $\mathcal{O}(4 - 14) = [0, 2, 3, 7]$ (the minor triad with added 9th). I have adapted the original notation somewhat, since that helps me in defining mathematical and musical operations conveniently: the label $\mathcal{O}$ stands for *original*, with pitch number $p_1 = 0 = c$ as the root or base of the structure.

   Next, I will aply the two basic operations *transposition* and *inversion* to the original set. Remember that the chromatic scale is *cyclic* in the sense that the 12 pitches will repeat at the second octave and so forth; the mathematical equivalent is the *modulo 12* operation. The notation for *translation* is $\mathcal{T}$,

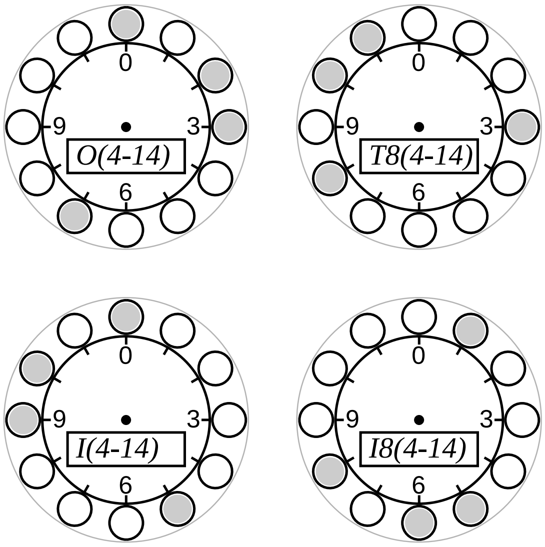

Figure 1: Pitch-class set diagrams example. Upper left: the original set $\mathcal{O}(4-14)$, upper right: the transposed set $\mathcal{T}_8(4-14)$, lower left: the inverted set $\mathcal{I}(4-14)$, lower right: the inverted and tranposed set $\mathcal{I}_8(4-14)$.

where i indicates the transposed root of the structure (thus $\mathcal{T}_0 = \mathcal{O}$). For example, $\mathcal{T}_8(4-14) = [8, 10, 11, 3]$. The notation for *inversion* is $\mathcal{I}_j$, where j indicates the root of the structure. For example, $\mathcal{I}_0(4-14) = [0, 10, 9, 5]$ and $\mathcal{I}_8(4-14) = [8, 6, 5, 1]$.

2. The *Disc Diagram*. The cyclic or modulo 12 property of the chromatic scale yields a convenient tool for graphic representation: the disc diagram. This was also discovered by the Dutch composer Peter Schat, who called it *"de Toonklok"* (the tone clock dial). Displaying the pitch-class sets on such a disc helps in visualising the interval content of the set. In the Prime Form system the interval content is presented as an *Interval Vector*: this is row vector with 6 elements $[n_1 n_2 n_3 n_4 n_5 n_6]$ (no spaces between elements), with n_i indicating the total number of interval classes with i semitones ($i = 1$ is the semitone, $i = 2$ the major second or whole tone, and so forth until $i = 6$ the augmented 4th or diminished 5th). So, for example, we find $(4-14) = [111120]$. Note the difference with the notation for the set itself: a comma-separated list.

   When using the disc diagram we mark the appropriate positions on the dial. See the example with set $\mathcal{O}(4-14)$ in Figure 1 (upper left). After some experience, the graphical image of the disc will give us an easier overview of pitch and interval content of the pitch-class set. Also a number of operations will become easier: translation of the set implies a *rotation* of the disc, while inversion is equivalent to *flipping* the disc (printing the disc on transparent material would be most helpful). This is also illustrated in Figure 1.

   Another frequent operation is the determination of the overlap between two, not necessarily equal sets, after translation. This can be done by putting two discs on top of each other, rotating them and then identify the marked digits on the disc.

   The discs may also be used for 12-tone series; put the sequence of pitches at the appropriate dial positions (do not colour or mark them, but put p_i at the positions 0 to 11).

3 The disc diagrams

Figure 2 and 3 contain a number of pitch-class set diagrams at two sizes. The smaller discs fit inside the larger discs.

Guidelines for using these diagrams are:

- Print the page with blank disc diagrams on thick paper or transparent material. Cut out the individual discs.
- Color the appropriate digits on the disc (small circles on the outer edge) and write the label in the central rectangular box, e.g., $\mathcal{O}(4-14)$. In case of an inverted set, indicate the inversion $\mathcal{I}_0(4-14)$. When defining a 12-tone series put the pitch numbers p_1 to p_{12} in the small circles.
- Push a pin or needle through the centre and fix them on a piece of foam.
- Before applying operations to the set or tone row, copy the original set on a small and large disc (or the inverted set). Now start to rotate the two discs relatively to each other and inspect the properties (overlap, complement, invariance, etc.)

References

[1] Jonathan Dunsby and Arnold Whittall, *Musical Analysis in Theory and Practice*, Faber Music, London, ISBN 0-571-10065-1, 1988.

[2] Allen Forte, *The Structure of Atonal Music*, Yale University Press, New Haven and London, ISBN 0-300-02120-8, 1973.

[3] Joel Lester, *Analytic Approaches to Twentieth-Century Music*, W.W. Norton & Company, Inc., New York, ISBN 0-393-95762-4, 1989.

[4] George Perle, *Serial Composition and Atonality: an Introduction to the Music of Schoenberg, Berg and Webern, 6th Edition, Revised*, University of California Press, Berkeley and Los Angeles, CA, ISBN 0-520-07430-0, 1991.

[5] George Perle, *Twelve-Tone Tonality, 2nd Edition, Revised and Expanded*, University of California Press, Berkeley and Los Angeles, CA, ISBN 0-520-20142-6, 1996.

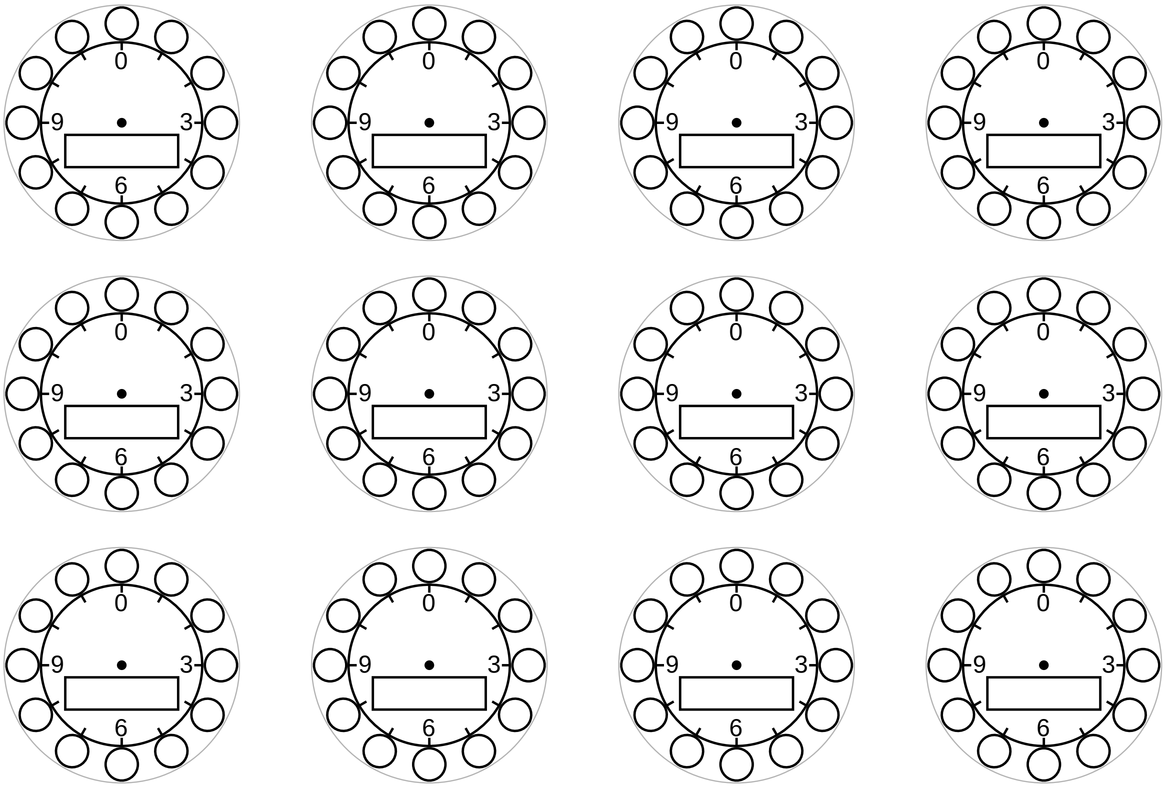

Figure 2: Pitch-class set diagrams (inner disc).

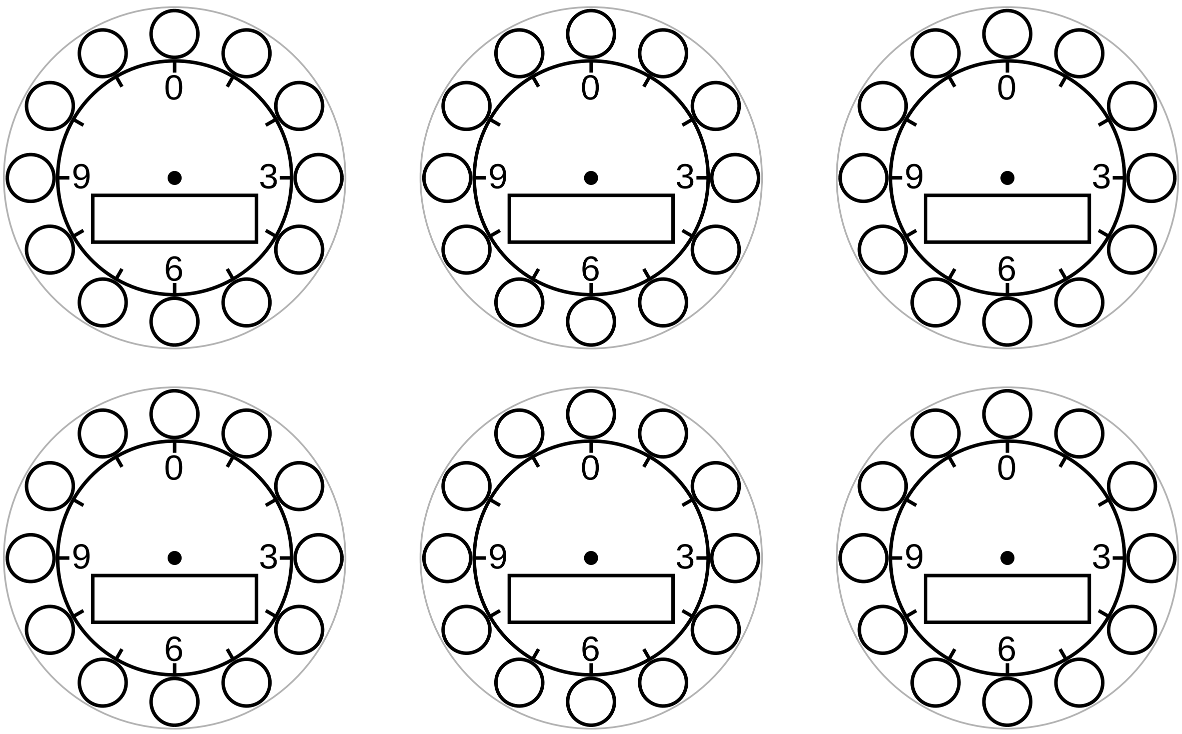

Figure 3: Pitch-class set diagrams (outer disc).